Latin America's Declining Skill Premium: A Macroeconomic Analysis

Technical Appendix

Juan F. Guerra-Salas[†]

Central Bank of Chile

April 2017

Note: This appendix is not a self-contained document.

Contents

A DSGE Model: System of Expectational Difference Equations

The following system of 32 equations in 32 variables describes the competitive equilibrium of the DSGE model.

$$(n_{H,t})^{\omega-1} = (1-\tau)\alpha_x \frac{Y_t^x}{N_{H,t}^x}\frac{1}{p_{H,t}} \tag{E.1}$$

$$(n_{H,t})^{\omega-1} = (1-\tau)p_{z,t}\alpha_z \frac{Y_t^z}{N_{H,t}^z}\frac{1}{p_{H,t}} \tag{E.2}$$

[†]Research Division, Central Bank of Chile. Address: Agustinas 1180, Santiago, Chile. Email: jguerra@bcentral.cl.

$$(n_{L,t})^{\omega-1} = (1-\tau)(1-\alpha_x)\frac{Y_t^x}{N_{L,t}^x}\frac{1}{p_{L,t}} \tag{E.3}$$

$$(n_{L,t})^{\omega-1} = (1-\tau)p_{z,t}(1-\alpha_z)\frac{Y_t^z}{N_{L,t}^z}\frac{1}{p_{L,t}} \tag{E.4}$$

$$n_{H,t} = N_{H,t}^x + N_{H,t}^z \tag{E.5}$$

$$n_{L,t} = N_{L,t}^x + N_{L,t}^z \tag{E.6}$$

$$\left[c_{H,t} - \frac{1}{\omega}(n_{H,t})^{\omega}\right]^{-\sigma} = \beta\mathbb{E}_t\left\{\left[c_{H,t+1} - \frac{1}{\omega}(n_{H,t+1})^{\omega}\right]^{-\sigma}\frac{p_{H,t}}{p_{H,t+1}}(1+r_{H,t+1})\right\} \tag{E.7}$$

$$\left[c_{L,t} - \frac{1}{\omega}(n_{L,t})^{\omega}\right]^{-\sigma} = \beta\mathbb{E}_t\left\{\left[c_{L,t+1} - \frac{1}{\omega}(n_{L,t+1})^{\omega}\right]^{-\sigma}\frac{p_{L,t}}{p_{L,t+1}}(1+r_{L,t+1})\right\} \tag{E.8}$$

$$c_{H,t}^z = \varphi\left(\frac{p_{z,t}}{p_{H,t}}\right)^{-\chi}c_{H,t} \tag{E.9}$$

$$c_{L,t}^z = \varphi\left(\frac{p_{z,t}}{p_{L,t}}\right)^{-\chi}c_{L,t} \tag{E.10}$$

$$c_{H,t}^x = (1-\varphi)\left(\frac{1}{p_{H,t}}\right)^{-\chi}c_{H,t} \tag{E.11}$$

$$c_{L,t}^x = (1-\varphi)\left(\frac{1}{p_{L,t}}\right)^{-\chi}c_{L,t} \tag{E.12}$$

$$C_t = c_{H,t} + c_{L,t} \tag{E.13}$$

$$C_t^z = c_{H,t}^z + c_{L,t}^z \tag{E.14}$$

$$C_t^x = c_{H,t}^x + c_{L,t}^x \tag{E.15}$$

$$G_t = \bar{G} + \phi_G(Y_{t-1} - \bar{Y}) \tag{E.16}$$

$$T_t = \phi_T(D_{t-k}^G - \bar{D^G}) \tag{E.17}$$

$$D_{t+1}^G = p_t^G G_t + (1+r)D_t^G - \left[\tau(Y_t^x + p_{z,t}Y_t^z) + T_t + p_t^{co}\bar{Co}\right] \tag{E.18}$$

$$G_t^z = \varphi \left(\frac{p_{z,t}}{p_t^G} \right)^{-\chi} G_t \tag{E.19}$$

$$G_t^x = (1-\varphi) \left(\frac{1}{p_t^G} \right)^{-\chi} G_t \tag{E.20}$$

$$\ln(p_t^{co}) = (1-\rho_{co})\ln(p^{\bar{co}}) + \rho_{co}\ln(p_{t-1}^{co}) + \epsilon_t^{co} \tag{E.21}$$

$$Y_t^x = A^x \left(N_{H,t}^x \right)^{\alpha_x} \left(N_{L,t}^x \right)^{1-\alpha_x} \tag{E.22}$$

$$Y_t^z = A^z \left(N_{H,t}^z \right)^{\alpha_z} \left(N_{L,t}^z \right)^{1-\alpha_z} \tag{E.23}$$

$$r_{H,t} = r + z_t^r + \psi(e^{d_{H,t}-d_H} - 1) \tag{E.24}$$

$$r_{L,t} = r + z_t^r + \psi(e^{d_{L,t}-d_L} - 1) \tag{E.25}$$

$$\ln(z_t^r) = (1-\rho_{zr})\ln(\bar{z^r}) + \rho_{zr}\ln(z_{t-1}^r) + \epsilon_t^{zr} \tag{E.26}$$

$$Y_t^z = C_t^z + G_t^z \tag{E.27}$$

$$c_{H,t}^x + p_{z,t}c_{H,t}^z + (1+r_{H,t})d_{H,t} = (1-\tau)\alpha_x \frac{Y_t^x}{N_{H,t}^x} n_{H,t} + d_{H,t+1} - T_t \tag{E.28}$$

$$c_{L,t}^x + p_{z,t}c_{L,t}^z + (1+r_{L,t})d_{L,t} = (1-\tau)(1-\alpha_x) \frac{Y_t^x}{N_{L,t}^x} n_{L,t} + d_{L,t+1} - T_t \tag{E.29}$$

$$p_{H,t} = \left[\varphi \left(p_{z,t} \right)^{1-\chi} + (1-\varphi) \right]^{\frac{1}{1-\chi}} \tag{E.30}$$

$$p_{L,t} = \left[\varphi \left(p_{z,t} \right)^{1-\chi} + (1-\varphi) \right]^{\frac{1}{1-\chi}} \tag{E.31}$$

$$p_t^G = \left[\varphi \left(p_{z,t} \right)^{1-\chi} + (1-\varphi) \right]^{\frac{1}{1-\chi}} \tag{E.32}$$

B DSGE Model: The Deterministic Steady State

Consider the system of expectational difference equations evaluated at the steady state, in which shocks are held at their mean value and variables are constant. The following set of equations describes the steady state of the model, where variables without time subscripts

denote steady state values.[1]

$$\beta = \frac{1}{1 + r + \bar{z^r}} \tag{S.1}$$

$$\frac{N_H^z}{N_L^z} = \left[\left(\frac{1 - \alpha_x}{1 - \alpha_z} \right) \left(\frac{\alpha_z}{\alpha_x} \right)^{\frac{\alpha_x}{\alpha_x - 1}} \right]^{\frac{\alpha_x - 1}{\alpha_x - \alpha_z}} \tag{S.2}$$

$$\frac{N_H^x}{N_L^x} = \left[\frac{\alpha_z}{\alpha_x} \left(\frac{N_H^z}{N_L^z} \right)^{\alpha_z - 1} \right]^{\frac{1}{\alpha_x - 1}} \tag{S.3}$$

$$\tau = \frac{(S_G + (r + \bar{z^r}) S_{Dg} - S_{CO})}{1 - S_{CO}} \tag{S.4}$$

$$N_H = \left[(1 - \tau) \alpha_x \left(\frac{N_H^x}{N_L^x} \right)^{\alpha_x - 1} \right]^{\frac{1}{\omega - 1}} \tag{S.5}$$

$$N_L = \left[(1 - \tau)(1 - \alpha_x) \left(\frac{N_H^x}{N_L^x} \right)^{\alpha_x} \right]^{\frac{1}{\omega - 1}} \tag{S.6}$$

$$N_L^z = \frac{N_H - \left(\frac{N_H^x}{N_L^x} \right) N_L}{\left(\frac{N_H^z}{N_L^z} - \frac{N_H^x}{N_L^x} \right)} \tag{S.7}$$

$$N_H^z = \left(\frac{N_H^z}{N_L^z} \right) N_L^z \tag{S.8}$$

$$N_H^x = N_H - N_H^z \tag{S.9}$$

$$N_L^x = N_L - N_L^z \tag{S.10}$$

$$Y^x = (N_H^x)^{\alpha_x} (N_L^x)^{1 - \alpha_x} \tag{S.11}$$

$$Y^z = (N_H^z)^{\alpha_z} (N_L^z)^{1 - \alpha_z} \tag{S.12}$$

$$Y = \frac{(Y^x + Y^z)}{1 - S_{CO}} \tag{S.13}$$

$$\bar{Co} = S_{CO} Y \tag{S.14}$$

[1] I normalize the steady state value of the following variables: $A^z, A^x, p_z, p_H, p_L, p_G, p^{co}$. Their value is one in the steady state.

$$G = S_G Y \tag{S.15}$$

$$D^G = S_{Dg} Y \tag{S.16}$$

$$D = S_D Y \tag{S.17}$$

$$w_H = (1-\tau)\alpha_x \frac{Y^x}{N_H^x} \tag{S.18}$$

$$w_L = (1-\tau)(1-\alpha_x)\frac{Y^x}{N_L^x} \tag{S.19}$$

$$c_H = w_H N_H - (r + \bar{z^r})(D/2) \tag{S.20}$$

$$c_L = w_L N_L - (r + \bar{z^r})(D/2) \tag{S.21}$$

$$C = c_H + c_L \tag{S.22}$$

$$\varphi = \frac{Y^z}{C+G} \tag{S.23}$$

$$c_H^z = \varphi c_H \tag{S.24}$$

$$c_H^x = (1-\varphi) c_H \tag{S.25}$$

$$c_L^z = \varphi c_L \tag{S.26}$$

$$c_L^x = (1-\varphi) c_L \tag{S.27}$$

$$G^z = \varphi G \tag{S.28}$$

$$G^x = (1-\varphi) G \tag{S.29}$$

$$C^z = c_H^z + c_L^z \tag{S.30}$$

$$C^x = c_H^x + c_L^x \tag{S.31}$$

Table 1: Panel Regressions of the Gini Index (net income) on the Output Gap

Dependent variable: Gini index (*net* income)			
Variable	**(a) 1970-2013**	**(b) 1970-2000**	**(c) 2001-2013**
Intercept	47.90***	48.13***	47.36***
	(0.25)	(0.23)	(0.45)
Output gap	-0.02	0.04	-0.26***
	(0.04)	(0.03)	(0.06)
Adj. R^2 within	0.60	0.69	0.63
H_0: $\mu_i = 0$	F(13, 439)=52.58***	F(13, 273)=49.76***	F(13, 151)=20.86***
Obs (unbalanced)	454	288	166

Robust White cross-section standard errors in parentheses; *** denotes significance at the 1% level. Unbalanced panel regressions of the Gini index (in levels) on HP-filtered real GDP with country fixed effects. See the note to figure 1 in the paper for the list of 14 countries considered.

C Robustness Checks: Income Inequality Over the Business Cycle

This section documents the robustness of the first piece of evidence presented in the paper—that income inequality is acyclical prior to the 2000s, but countercyclical thereafter. Table 1 shows regressions in which the dependent variable is the Gini index on *net* income (as opposed to market income). Table 2 shows regressions in which the year the second-subsample begins changes. Finally, in table 3, regressions are based on balanced panels. In all three robustness exercises, the main findings of the paper hold.

D Robustness Checks: Sectoral Composition of Economic Expansions

This section checks the robustness of the second piece of evidence presented in the paper—that the boom of the 2000s was concentrated on service sectors, regardless of the date one chooses as the beginning of the expansion. Figure 1 shows the cumulative growth in value added by sector for two alternative periods: 2000–2010, and 2002–2010. Clearly, the main findings of the paper hold.

Table 2: Panel Regressions of the Gini Index on the Output Gap (alternative sub-samples)

Dependent variable: Gini index (market income)				
Variable	**(a) 1999-2013**	**(b) 2000-2013**	**(c) 2002-2013**	**(c) 2003-2013**
Intercept	50.69*** (0.45)	50.51*** (0.45)	50.09*** (0.42)	49.89*** (0.42)
Output gap	-0.17*** (0.06)	-0.20*** (0.06)	-0.24*** (0.07)	-0.21** (0.09)
Adj. R^2 within	0.60	0.62	0.72	
H_0: $\mu_i = 0$	F(13,179)=21.83***	F(13,165)=22.64***	F(13,137)=20.02***	F(13,123)=27.20***
Obs (unbalanced)	194	180	152	138

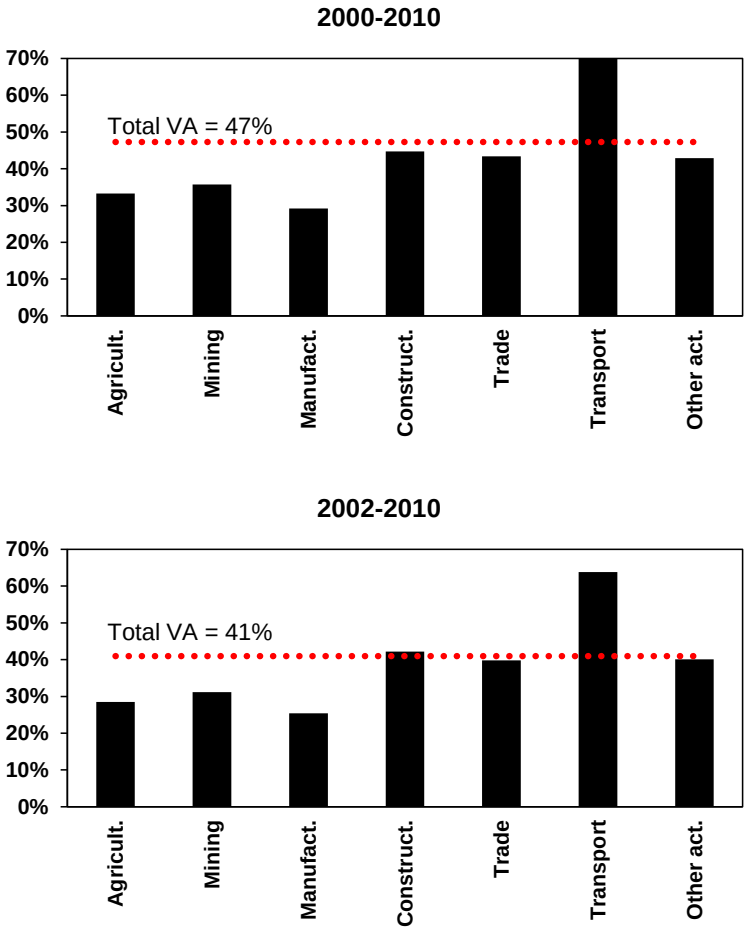
Robust White cross-section standard errors in parentheses; *** denotes significance at the 1% level. Unbalanced panel regressions of the Gini index (in levels) on HP-filtered real GDP with country fixed effects. See the note to figure 1 in the paper for the list of 14 countries considered.

Table 3: Panel Regressions of the Gini Index on the Output Gap (balanced sample)

Dependent variable: Gini index (market income)			
Variable	**(a) 1990-2011**	**(b) 1990-2000**	**(c) 2001-2011**
Intercept	51.07*** (0.32)	51.44*** (0.35)	50.55*** (0.44)
Output gap	-0.02 (0.05)	0.07* (0.04)	-0.21*** (0.05)
Adj. R^2 within	0.61	0.71	0.68
Obs (balanced)	308	154	154

Robust White cross-section standard errors in parentheses; *** denotes significance at the 1% level. Unbalanced panel regressions of the Gini index (in levels) on HP-filtered real GDP with country fixed effects. See the note to figure 1 in the paper for the list of 14 countries considered.

Figure 1: Cumulative Growth in Value Added (alternative dates)



Median cumulative growth of value added across 14 countries. Cumulative growth is the percent difference between the final and initial year. The underlying data is expressed in constant 2005 prices and domestic currency. Sectoral groupings based on ISIC 3.0 classification of economic activities: Agriculture, hunting, forestry, fishing (ISIC A-B); Mining, manufacturing, utilities (ISIC C-E); Manufacturing (ISIC D); Construction (ISIC F); Wholesale, retail trade, restaurants and hotels (ISIC G-H); Transport, storage and communication (ISIC I); Other activities (ISIC J-P). See the note to figure 1 in the paper for the list of countries considered.

Source: Author's calculations based on data from the United Nations Statistics Division, March 2017.