Search Frictions and the Business Cycle in a Small Open Economy DSGE Model*

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Abstract

A labor market specification featuring search frictions and endogenous separations within a standard New Keynesian small open economy model significantly improves its ability to explain and predict both labor market data and other macroeconomic variables. We estimate the model with Chilean data and find that variations along the extensive margin of labor supply (i.e. employment) play a crucial role in the propagation of shocks, whereas the intensive margin (i.e. hours) is not important. Furthermore, foreign shocks and shocks to trend growth are key drivers of the business cycle, which is consistent with the empirical literature on open emerging economies. We conclude that a medium-scale DSGE model with this richer labor market specification is superior to one featuring the standard assumption of a labor market that always clears at a sticky nominal wage (à la Calvo) through variations along the intensive margin.

Keywords: Labor market; Search and matching; DSGE models; Business cycles; Small open economies.

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1 Introduction

Most New Keynesian DSGE models used for policy analysis and forecasting at central banks and other policy institutions assume that the labor market always clears at a sticky nominal wage (à la Calvo) through variations along the intensive margin of labor supply (i.e. hours), but there is no role for adjustment along the extensive margin (i.e. employment).[1] The latter stands in stark contrast to academic research that has emphasized the role of labor market flows based on search and matching theory. According to that literature, search frictions and matching can successfully explain several relevant labor market facts such as the existence of involuntary unemployment and the dynamics of job creation and job destruction (see Pissarides, 2011).[2]

Some of that disconnect between labor market research and labor market specifications in practical policy models may be due to the fact that the usefulness of search frictions in medium-scale quantitative DSGE models for monetary policy analysis and forecasting is not yet sufficiently well understood. Hence, in this paper we let search frictions compete with the standard labor market specification, which we call "Calvo wages", in a DSGE framework. In particular, we assess whether and how the inclusion of a search and matching specification à la Diamond (1982), Mortensen (1982) and Pissarides (1985) with both margins of labor supply and endogenous separations following Mortensen and Pissarides (1994), Cooley and Quadrini (1999) and den Haan, Ramey, and Watson (2000) improves the empirical fit and forecasting performance of an otherwise standard New Keynesian small open economy (NK-SOE) model. The analysis is conducted using Bayesian techniques and Chilean data. While our paper forms part of several recent studies that have investigated the usefulness of labor market search and matching in macroeconomic models, as we discuss below, we are among the first to analyze the benefits of search frictions in a small open economy context. In addition, only few related studies have examined the relevance of endogenous separations with both margins of labor supply in estimated DSGE models. As we show, the latter has several relevant implications for the dynamics of our model.

The shortcomings of labor market specifications in standard DSGE models, both for closed and open economies, become clear from a brief review. In particular, exogenous labor market shocks are typically found to be important drivers of aggregate dynamics in those models: in

[1] Examples of DSGE models used at central banks and other policy institutions that incorporate Calvo-type wage stickiness or some other form of wage stickiness that gives rise to a wage Phillips curve, e.g. due to wage adjustment costs à la Rotemberg, are discussed in Brubakk and Sveen (2009), Burgess et al. (2013), Chung, Kiley, and Laforte (2010), de Castro, Gouvea, Minella, dos Santos, and Souza-Sobrinho (2011), Del Negro et al. (2013), Dorich, Johnston, Mendes, Murchison, and Zhang (2013), Erceg, Guerrieri, and Gust (2006), González, Mahadeva, Prada, and Rodríguez (2011), Lees (2009), Medina and Soto (2007), Ratto, Roeger, and in 't Veld (2009), Schorfheide, Sill, and Kryshko (2010), and Smets, Christoffel, Coenen, Motto, and Rostagno (2010).

[2] See also Krause and Lubik (2014).

the Smets and Wouters (2003) euro area model, labor supply preference shocks are the most important drivers of output while wage markup shocks are responsible for the bulk of variations in real wages; whereas in the Smets and Wouters (2007) U.S. model where there is no separate labor supply shock, wage markup shocks account for most of medium- to long-term fluctuations in output and inflation. In Adolfson, Laséen, Lindé, and Villani's (2007) NK-SOE model, labor supply shocks are also among the most important shocks to explain output, wage and inflation dynamics in Sweden. The fact that exogenous labor market shocks are so important in standard DSGE models seems unsatisfactory, not only because their underlying determinants are hard to identify, but also because one might expect that labor market outcomes are to a large extent consequences of more structural shocks such as monetary or fiscal policy shocks or, in open economies, foreign shocks (i.e. shocks to foreign interest rates, foreign demand, commodity prices, etc.). In addition, all of the above models rely on relatively large real wage elasticities of individual hours worked to match fluctuations in total hours, which is known to be at odds with micro evidence (see Chetty, Guren, Manoli, and Weber, 2011).

Due to the above shortcomings, recent model developments using search and matching theory have attempted to improve labor market specifications and generate stronger endogenous propagation properties of DSGE models. For instance, Christiano, Trabandt, and Walentin (2011) describe the labor market in a NK-SOE model using a search and matching framework with variations on both margins of labor supply.[3] Their estimation results for Swedish data show that the labor supply shock becomes unimportant in explaining output, the estimated elasticity of individual hours is relatively low, and no wage markup shock is needed. However, the labor supply shock is still the most important shock for both total hours and real wages, and basic foreign shocks are relatively unimportant for aggregate dynamics.[4] An earlier study by Krause, Lopez-Salido, and Lubik (2008) based on a closed economy model with search and matching estimated with U.S. data also found a relatively low elasticity of individual hours and a small role for labor supply shocks. However, in this model price markup and (ad hoc) match efficiency shocks are the dominant force in labor market fluctuations.[5] Part of the failure of this model to explain the fluctuations of both labor market variables and other macroeconomic variables such as output and inflation through more structural shocks may be due to the absence of endogenous separations, in line with the results of Sedláçek (2014). Indeed, many studies have found that endogenous separations are important for understanding labor market flows

[3] See also Adolfson, Laséen, Christiano, Trabandt, and Walentin (2013).

[4] This is related to the problem that NK-SOE models tend to have difficulties in accounting for the substantial influence of foreign shocks identified in many time series studies (see Justiniano and Preston, 2010).

[5] Similar results were obtained by Albertini, Kamber, and Kirker (2012) in a New Keynesian small open economy model estimated with data for New Zealand.

and their interaction with output and inflation (e.g. Trigari, 2009).[6]

Hence, the success of search frictions in quantitative DSGE models has so far been mixed. Our results shed additional light on this issue. In particular, we find that the data strongly favor the model with search frictions over the model with Calvo wages, as reflected by a significantly higher marginal data density, as well as a significantly better ability of the model with search frictions to match the majority of the second moments of the data. Furthermore, in the model with search frictions, foreign shocks and shocks to trend growth (i.e., permanent TFP shocks) are far more important drivers of the business cycle than in the model with Calvo wages. This finding is consistent with the large literature on the drivers of business cycles in small emerging economies such as Chile.[7] Finally, the forecasting performance of the model, for labor market variables as well as other key variables such as output and inflation, is also significantly improved by the search frictions. Our paper therefore provides further evidence that search frictions are useful to explain aggregate dynamics in quantitative DSGE models for small open economies.

The rest of the paper is organized as follows. Section 2 presents our NK-SOE model with search frictions and matching, while the variant of the model with Calvo wages is described in Section 3.[8] Section 4 describes the calibration and estimation strategy, while Section 5 compares the fit of the model under the two labor market specifications, discusses the role of search frictions in propagating various types of shocks, and provides an analysis of the forecasting performance of the different models. Finally, section 6 concludes.

2 An NK-SOE Model with Search and Matching

This section presents our NK-SOE model with nominal and real rigidities, and search and matching à la Diamond (1982), Mortensen (1982) and Pissarides (1985) with endogenous separations in the labor market, following Cooley and Quadrini (1999) and den Haan et al. (2000). The core model shares the structure of the baseline NK-SOE model presented in García-Cicco, Kirchner, and Justel (2015).[9] Domestic goods are produced with capital and labor, there is habit formation in consumption, there are adjustment costs in investment, firms face a Calvo-pricing problem with partial indexation, and there is imperfect exchange rate pass-through into import prices in the short run due to local currency price stickiness. The economy also exports an exogenous

[6] A few recent studies have investigated the implications of search frictions in an emerging market context, including Boz, Durdu, and Li (2015) and Medina and Naudon (2012). However, these studies are based on calibrated models that abstract from nominal rigidities as well as the intensive margin and endogenous separations, unlike in our paper.

[7] See, for example, Fernández, Schmitt-Grohé, and Uribe (2017), Fernández, González, and Rodríguez (2017), Kose (2002), Mendoza (1995), and Aguiar and Gopinath (2007).

[8] The equilibrium conditions and the strategy for the steady state for both models are provided in the appendix.

[9] The model is a shrinked-down version of the Medina and Soto (2007) model used for policy analysis and forecasting at the Central Bank of Chile.

endowment of a commodity good, which captures the importance of commodity exports in many small open economies including Chile. The economy is subject to shocks to preferences, the rate of separation from employment, technology (neutral and investment-specific), commodity production, government expenditures, monetary policy, foreign demand, foreign inflation, foreign interest rates, and the international price of the commodity good.

2.1 Households

There is a continuum of infinitely lived households normalized to one with identical asset endowments and identical preferences. Household members, part of a continuum of mass one, can be either employed or unemployed. All members pool their assets so as to ensure equal consumption, that is, there is perfect insurance of unemployment risk. Each member has the following separable utility function with habit formation:[10]

$$u(C_t, \check{C}_{t-1}) - g(h_t) = \frac{1}{1-\sigma}\left[\left(C_t - \varsigma \check{C}_{t-1}\right)^{1-\sigma} - 1\right] - \Theta_t A_{t-1}^{1-\sigma} \kappa_t \frac{h_t^{1+\phi}}{1+\phi},$$

where C_t is individual consumption of a final good, $\check{C}_t$ is aggregate consumption (which each member takes as given), h_t is hours per worker, κ_t is an exogenous shock to the disutility of labor supply, A_t is an economy-wide stochastic trend (see below), and Θ_t is an endogenous preference shifter that satisfies[11,12,13]

$$\Theta_t \equiv \widetilde{\chi}_t A_{t-1}^{\sigma} (\check{C}_t - \varsigma \check{C}_{t-1})^{-\sigma}, \qquad \widetilde{\chi}_t = \widetilde{\chi}_{t-1}^{1-\upsilon} A_{t-1}^{-\sigma\upsilon} \left(\check{C}_t - \varsigma \check{C}_{t-1}\right)^{\sigma\upsilon}.$$

The parameters σ, ϕ and ς are the inverse elasticity of intertemporal substitution, the inverse Frisch elasticity of hours worked, and the degree of habit formation, respectively. The welfare function of a representative household over time is then given by[14]

$$E_t \sum_{s=0}^{\infty} \beta^s \varrho_{t+s} \left[u\left(C_{t+s}, \check{C}_{t+s-1}\right) - n_{t+s} g\left(h_{t+s}\right)\right], \tag{1}$$

[10]Throughout, uppercase letters denote variables containing a unit root in equilibrium (either due to technology or to long-run inflation) while lowercase letters indicate variables with no unit root. Real variables are constructed using the domestic consumption good as the numeraire. In the appendix we describe how each variable is transformed to achieve stationarity in equilibrium. Variables without time subscript denote non-stochastic steady state values in the stationary model.

[11]We assume external habit formation instead of internal habit formation as in García-Cicco, Kirchner, and Justel (2015) to simplify the analysis.

[12]The disutility of work is multiplied by $A_{t-1}^{1-\sigma}$ to maintain a balanced steady-state growth path.

[13]The formulation of this endogenous preference shifter follows Galí, Smets, and Wouters (2011).

[14]Under separable preferences and external habit formation, (1) results from the general specification $E_t \sum_{s=0}^{\infty} \beta^s \varrho_{t+s} \left[n_{t+s} u\left(C_{t+s}^n, \check{C}_{t+s-1}\right) - n_{t+s} g\left(h_{t+s}\right) + (1-n_{t+s}) u\left(C_{t+s}^u, \check{C}_{t+s-1}\right)\right]$ since, in equilibrium, $C_t^n = C_t^u$ for all t.

where $\beta \in (0,1)$ is the intertemporal discount factor, ϱ_t is an exogenous preference shock and n_t is the number of employed household members. Note that, in equilibrium, $C_t = \check{C}_t$ for all t.

Households save and borrow by purchasing domestic currency-denominated government bonds (B_t) and by trading foreign currency bonds (B_t^*) with foreign agents, both being non-state contingent assets. They also purchase an investment good (I_t) which determines next period's physical capital stock (K_t). Let r_t, r_t^* and r_t^K denote the gross real returns on B_{t-1}, B_{t-1}^* and K_t, respectively. The employed members earn a real wage of W_t per hour, while unemployed members earn an amount b_t^u of unemployment benefits, which is paid out by the government.[15] Let rer_t be the real exchange rate (i.e. the price of foreign consumption goods in terms of domestic consumption goods), let T_t denote real lump-sum tax payments to the government and let Σ_t collect real dividend income from the ownership of firms. The period-by-period budget constraint of the household is then given by

$$C_t + I_t + B_t + rer_t B_t^* + T_t = W_t h_t n_t + (1-n_t) b_t^u + r_t B_{t-1} + rer_t r_t^* B_{t-1}^* + r_t^K K_{t-1} + \Sigma_t. \quad (2)$$

The physical capital stock evolves according to the law of motion:

$$K_t = (1-\delta) K_{t-1} + [1 - \Gamma(I_t / I_{t-1})] \varpi_t I_t, \qquad \delta \in (0,1], \quad (3)$$

where I_t denotes investment expenditures and

$$\Gamma\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma}{2}\left(\frac{I_t}{I_{t-1}} - \bar{a}\right)^2, \qquad \gamma \geq 0, \qquad \bar{a} \geq 1,$$

are convex investment adjustment costs. The variable ϖ_t is an investment shock that captures changes in the efficiency of the investment process.[16] The household chooses C_t, I_t, K_t, B_t, and B_t^* to maximize (1) subject to (2)-(3), taking W_t, h_t, n_t, r_t, r_t^*, r_t^K, rer_t, T_t, Σ_t, B_{t-1}, B_{t-1}^* and K_{t-1} as given. The household's employment (n_t), hours worked (h_t), and the real wage (W_t) are determined as outcomes of a search and matching process and a bargaining process (see below).

The nominal interest rates are implicitly defined as

$$r_t = R_{t-1} \pi_t^{-1}, \qquad r_t^* = R_{t-1}^* \xi_{t-1} (\pi_t^*)^{-1},$$

where $\pi_t = P_t / P_{t-1}$ and $\pi_t^* = P_t^* / P_{t-1}^*$ denote the gross inflation rates of the domestic and foreign consumption-based price indices P_t and P_t^*, respectively. The variable ξ_{t-1} denotes a

[15] We allow unemployment benefits to grow proportionately with A_{t-1} in order to maintain a balanced steady-state growth path. Then, $b_t^u = \bar{b} A_{t-1}$ with $\bar{b} \geq 0$.

[16] See Greenwood, Hercowitz, and Krusell (1997) and Justiniano, Primiceri, and Tambalotti (2011).

country premium given by (see Schmitt-Grohé and Uribe, 2003):

$$\xi_t = \bar{\xi} \exp \left[-\psi \frac{rer_t B_t^*/A_{t-1} - rer \times b^*}{rer \times b^*} + \frac{\zeta_t^o - \zeta^o}{\zeta^o} + \frac{\zeta_t^u - \zeta^u}{\zeta^u} \right], \qquad \psi > 0, \qquad \bar{\xi} \geq 1,$$

where ζ_t^o and ζ_t^u are exogenous shocks to the country premium, where we assume that ζ_t^o is observable while ζ_t^u is unobservable. The foreign nominal interest rate R_t^* evolves exogenously, whereas the domestic central bank sets R_t.

2.2 Labor Market Dynamics

The labor market is subject to Diamond-Mortensen-Pissarides-type search frictions and matching. In order to form new employment relationships (matches), workers must search and firms must post vacancies. We assume that all unemployed workers look for jobs. The number of matches $\mathcal{M}_t$, which begin work in period $t+1$, is given by the matching function $\mathcal{M}_t = m\, v_t^{1-\mu} u_t^{\mu}$, where u_t is the number of searching workers, v_t is the number of vacancies posted, m is a match efficiency parameter, and $\mu \in (0,1)$ is the match elasticity. At the beginning of each period, before new matches are formed, a fraction ρ_t of existing matches terminate. The total separation rate $\rho_t = \rho_t^x + (1-\rho_t^x)\rho_t^n$ includes the exogenous component ρ_t^x and the endogenous component ρ_t^n. Endogenous separations occur if the firm's operating cost $\widetilde{c}_t$ is greater than an endogenously determined threshold $\underline{c}_t$.[17] This operating cost is assumed to be i.i.d. across firms and time with c.d.f. $F(\cdot)$. The endogenous separation rate is therefore $\rho_t^n = \Pr(\widetilde{c}_t > \underline{c}_t) = 1 - F(\underline{c}_t)$. The evolution of aggregate employment is thus given by $n_t = (1-\rho_t)[n_{t-1} + \mathcal{M}_{t-1}]$, and the number of unemployed workers searching for a job is $u_t = 1 - n_t$. The probability that a searching worker is matched to a new job is then $s_t = \mathcal{M}_t/u_t = m\, (v_t/u_t)^{1-\mu}$, the probability that a firm fills a vacancy is $e_t = \mathcal{M}_t/v_t = m(v_t/u_t)^{-\mu}$, and labor market tightness can be defined as $\theta = v_t/u_t$.

2.3 Firms

There are different types of firms that are all owned by the households. There is a set of perfectly competitive wholesale firms that produce different varieties of a home good with labor and capital as inputs, a set of monopolistically competitive retail firms that buy and re-sell those varieties, a set of monopolistically competitive importing firms, and three groups of perfectly competitive aggregators: one packing different varieties of the home good into a composite home good, one packing imported varieties into a composite foreign good, and another one that bundles the composite home and foreign goods to create a final good. This final good is purchased by

[17] We assume additive idiosyncratic operating costs as in Cooley and Quadrini (1999) to avoid excessive cross-sectional heterogeneity in hours per worker, which would result from a specification as in den Haan et al. (2000) with a multiplicative idiosyncratic productivity shock in the production function.

households (C_t, I_t) and the government (G_t).[18] In addition, there is a set of competitive firms producing a homogeneous commodity good that is exported abroad. A proportion of those commodity-exporting firms is owned by the government and the remaining proportion is owned by foreign agents. The total mass of firms in each sector is normalized to one. Throughout, we denote productions/supply with the letter Y and inputs/demand with X.

2.3.1 Final Goods

A representative final goods firm demands composite home and foreign goods in the amounts X_t^H and X_t^F, respectively, and combines them according to the technology

$$Y_t^C = \left[(1-o)^{\frac{1}{\eta}} \left(X_t^H\right)^{\frac{\eta-1}{\eta}} + o^{\frac{1}{\eta}} \left(X_t^F\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \qquad o \in (0,1), \qquad \eta > 0. \tag{4}$$

Let p_t^H and p_t^F denote the relative prices of X_t^H and X_t^F in terms of the final good. Subject to (4), the firm maximizes its profits $\Pi_t^C = Y_t^C - p_t^H X_t^H - p_t^F X_t^F$ over the input demands X_t^H and X_t^F taking p_t^H and p_t^F as given.

2.3.2 Home Composite Goods

A representative home composite goods firm demands home goods of all varieties $j \in [0,1]$ in amounts $X_t^H(j)$ and combines them according to the technology

$$Y_t^H = \left[\int_0^1 X_t^H(j)^{\frac{\epsilon_H - 1}{\epsilon_H}} dj\right]^{\frac{\epsilon_H}{\epsilon_H - 1}}, \qquad \epsilon_H > 0. \tag{5}$$

Let $p_t^H(j)$ denote the price of the good of variety j in terms of the home composite good. Subject to (5), the firm maximizes its profits $\Pi_t^H = p_t^H Y_t^H - \int_0^1 p_t^H p_t^H(j) X_t^H(j) dj$ over the input demands $X_t^H(j)$ taking the relative prices p_t^H and $p_t^H(j)$ as given, which yields the input demand functions

$$X_t^H(j) = p_t^H(j)^{-\epsilon_H} Y_t^H, \quad \text{for all } j. \tag{6}$$

2.3.3 Wholesale Goods of Variety j and the Job Creation Condition

Wholesale goods of variety j are produced according to the technology

$$Y_t^H(j) = z_t K_{t-1}(j)^{\alpha} [A_t n_t(j) h_t(j)]^{1-\alpha}, \qquad \alpha \in [0,1), \tag{7}$$

[18] The final good is also used to pay vacancy posting and operating costs.

where z_t is an exogenous stationary neutral technology shock, while A_t (with $a_t \equiv A_t/A_{t-1}$) is a non-stationary labor-augmenting technology trend, both common to all varieties. Wholesale firms choose how much capital to rent and how much labor to hire, subject to an identical vacancy posting cost of ω_t per vacancy and firm-specific operating cost per worker $\widetilde{c}_t(j)$ (these costs are assumed to be paid in terms of final goods). Letting $p_t^m(j)$ denote the relative price of wholesale good j in terms of the final good, firm j's profit is given by

$$\Pi_t^m(j) = p_t^m(j)Y_t^H(j) - r_t^K K_{t-1}(j) - W_t h_t(j) n_t(j) - \mathcal{C}_t(j) - \mathcal{L}_t(j),$$

where

$$\mathcal{C}_t(j) = n_t(j) A_{t-1}\kappa_c \int_0^{\underline{c}_t(j)} \widetilde{c}_t(j) \frac{dF(\widetilde{c}_t(j))}{F(\underline{c}_t(j))} = n_t(j) H(\underline{c}_t(j))$$

is the total operating cost of firm j conditional on working, with $\kappa_c \geq 0$, while $\mathcal{L}_t(j) = \omega_t v_t(j)$ is the total vacancy posting cost with $\omega_t = \omega A_{t-1}$, $\omega \geq 0$.[19] The firm's workforce evolves over time as the number of workers from the previous period plus new hires, whose jobs do not get terminated:

$$n_t(j) = (1-\rho_t)(n_{t-1}(j) + e_{t-1}v_{t-1}(j)). \quad (8)$$

Since today's choice of $v_t(j)$ affects tomorrow's workforce, the firm faces an intertemporal decision problem to maximize expected discounted profits. Hence, the firm chooses $K_{t-1}(j)$, $n_t(j)$ and $v_t(j)$ to maximize $E_t \sum_{s=0}^{\infty} \Xi_{t,t+s} \Pi_{t+s}^m(j)$ subject to (7) and (8). The first-order conditions for this problem yield the job creation condition:[20]

$$\frac{\omega_t}{e_t} = E_t \Xi_{t,t+1} (1-\rho_{t+1}) \left(p_{t+1}^m mpn_{t+1} - H^C(\underline{c}_{t+1}) - W_{t+1}h_{t+1} + \frac{\omega_{t+1}}{e_{t+1}} \right).$$

That is, firms post vacancies to expand employment until the effective cost of posting an additional vacancy (ω_t times the expected duration of the vacancy $1/e_t$) equals the expected marginal product of an extra worker, minus wage payment and average operating cost, plus its expected return from the reduction of vacancy posting costs, conditional on the job not being severed in period $t+1$.

2.3.4 Retail Goods of Variety j

Retail firms buy and distribute wholesale goods. There is one retailer associated with each variety of the wholesale good. The retailer distributing variety j satisfies the demand given

[19] We allow operating costs and vacancy posting costs to grow proportionately with the technology trend to maintain a balanced steady-state growth path.

[20] We drop subscripts j due to symmetry.

by (6) but it has monopoly power for its variety. Given nominal marginal costs $P_t^H mc_t^H(j) = P_t p_t^m(j) = P_t p_t^m$, the firm chooses its price $P_t^H(j)$ to maximize profits.[21] In setting prices, the firm faces a Calvo-type problem, whereby each period it can change its price optimally with probability $1-\theta_H$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\vartheta_H \in [0,1]$ and $1-\vartheta_H$.

2.3.5 Foreign Composite Goods

A representative foreign composite goods firm demands foreign goods of all varieties $j \in [0,1]$ in amounts $X_t^F(j)$ and combines them according to the technology

$$Y_t^F = \left[\int_0^1 X_t^F(j)^{\frac{\epsilon_F - 1}{\epsilon_F}} dj\right]^{\frac{\epsilon_F}{\epsilon_F - 1}}, \qquad \epsilon_F > 0. \tag{9}$$

Let $p_t^F(j)$ denote the price of the good of variety j in terms of the foreign composite good. Subject to (9), the firm maximizes its profits $\Pi_t^F = p_t^F Y_t^F - \int_0^1 p_t^F p_t^F(j) X_t^F(j) dj$ over the input demands $X_t^F(j)$ taking the relative prices p_t^F and $p_t^F(j)$ as given. The first-order conditions yield the input demand functions:

$$X_t^F(j) = p_t^F(j)^{-\epsilon_F} Y_t^F, \quad \text{for all } j. \tag{10}$$

2.3.6 Foreign Goods of Variety j

Importing firms buy an amount M_t of a homogenous foreign good at the price P_t^{F*} in the world market and convert this good into varieties $Y_t^F(j)$ that are sold domestically, where $M_t = \int_0^1 Y_t^F(j) dj$. The firm producing variety j satisfies the demand given by (10) but it has monopoly power for its variety. As it takes one unit of the foreign good to produce one unit of variety j, nominal marginal costs in terms of composite goods prices are

$$P_t^F mc_t^F(j) = P_t^F mc_t^F = S_t P_t^{F*}, \tag{11}$$

where S_t is the nominal exchange rate (defined as the price of one unit of foreign currency in terms of domestic currency). Given marginal costs, the firm producing variety j chooses its price $P_t^F(j)$ to maximize profits. In setting prices, the firm faces a Calvo-type problem, whereby each period it can change its price optimally with probability $1-\theta_F$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady

[21] Note that $mc_t^H(j)$ is real marginal cost expressed in terms of home composite goods prices.

state inflation with weights $\vartheta_F \in [0,1]$ and $1-\vartheta_F$. In this way, the model features delayed pass-trough from international to domestic prices.

2.3.7 Commodities

A representative firm produces a quantity Y_t^{Co} of a commodity good in each period. Commodity production evolves exogenously according to the process

$$\log(Y_t^{Co}/A_{t-1}) = (1-\rho_{y^{Co}})\log(\bar{y}^{Co}) + \rho_{y^{Co}}\log(Y_{t-1}^{Co}/A_{t-2}) + \varepsilon_t^{y^{Co}}, \qquad \rho_{y^{Co}} \in [0,1), \qquad \bar{y}^{Co} > 0.$$

The entire production is sold abroad at a given international price P_t^{Co*}. The real foreign and domestic prices are denoted as p_t^{Co*} and p_t^{Co}, respectively, where p_t^{Co*} is assumed to evolve exogenously. The real domestic currency income generated in the commodity sector is therefore equal to $p_t^{Co}Y_t^{Co}$. The government receives a share $\chi \in [0,1]$ of this income and the remaining share goes to foreign agents.

2.4 Bargaining over Wages and Hours

Real wages and hours per worker are determined through Nash bargaining. On the firm side, the value of an open vacancy $\mathcal{V}_t^V$ is given by an exogenous vacancy posting cost ω_t, plus the discounted continuation values of filling the vacancy conditional on having the job not severed (with probability $e_t(1-\rho_{t+1})$), or an open vacancy in the next period:

$$\mathcal{V}_t^V = -\omega_t + E_t\Xi_{t,t+1}\left[e_t(1-\rho_{t+1})\int_0^{\underline{c}_t}\mathcal{V}_{t+1}^J(\widetilde{c}_{t+1})\frac{dF(\widetilde{c}_{t+1})}{F(\underline{c}_{t+1})} + (1-e_t(1-\rho_{t+1}))\mathcal{V}_{t+1}^V\right], \quad (12)$$

where $\Xi_{t,t+1}$ is the firm's stochastic discount factor for real payoffs.[22] The value of a filled job $\mathcal{V}_t^J$ given a draw $\widetilde{c}_t$ is equal to the firm's current-period profit generated by the worker (i.e. revenue minus production costs), plus the discounted continuation values of having the job severed next period (with probability ρ_{t+1}), or having it not severed (with probability $1-\rho_{t+1}$):

$$\mathcal{V}_t^J(\widetilde{c}_t) = p_t^m mpn_t - W_t^n(\widetilde{c}_t)h_t - A_{t-1}\kappa_c\widetilde{c}_t + E_t\Xi_{t,t+1}\left[\begin{array}{c}(1-\rho_{t+1})\int_0^{\underline{c}_{t+1}}\mathcal{V}_{t+1}^J(\widetilde{c}_{t+1})\frac{dF(\widetilde{c}_{t+1})}{F(\underline{c}_{t+1})}\\ +\rho_{t+1}\mathcal{V}_{t+1}^V\end{array}\right], \quad (13)$$

where p_t^m is the relative price of wholesale goods in terms of the final good, mpn_t is the marginal product of the worker and W_t^n is the negotiated wage. On the worker side, the value of being

[22] As the firms are owned by the households, the stochastic discount factor satisfies $\Xi_{t,t+s} \equiv \beta^s(\varrho_{t+s}/\varrho_t)(\Lambda_{t+s}/\Lambda_t)$, for $s \geq 0$.

employed in a job $\mathcal{V}_t^E$ with idiosyncratic operating cost $\widetilde{c}_t$ is equal to the worker's current-period benefit from the job (i.e. the wage payment minus the disutility from supplying hours expressed in terms of current consumption), plus the discounted continuation values of remaining on the job (with probability $1-\rho_{t+1}$) or being separated (with probability ρ_{t+1}):

$$\mathcal{V}_t^E(\widetilde{c}_t) = W_t^n(\widetilde{c}_t)h_t - \frac{g(h_t)}{\Lambda_t} + E_t\Xi_{t,t+1}\left[\begin{array}{c}(1-\rho_{t+1})\int_0^{\underline{c}_{t+1}}\mathcal{V}_{t+1}^E(\widetilde{c}_{t+1})\frac{dF(\widetilde{c}_{t+1})}{F(\underline{c}_{t+1})}\\ +\rho_{t+1}\mathcal{V}_{t+1}^U\end{array}\right], \tag{14}$$

where Λ_t is the household's marginal utility of consumption. The value of being unemployed $\mathcal{V}_t^U$ is equal to the current unemployed benefit, plus the discounted continuation values of finding a job conditional on having the match not severed next period with probability $s_t(1-\rho_{t+1})$ or, otherwise, remaining unemployed:

$$\mathcal{V}_t^U = b_t^u + E_t\Xi_{t,t+1}\left[s_t(1-\rho_{t+1})\int_0^{\underline{c}_{t+1}}\mathcal{V}_{t+1}^E(\widetilde{c}_{t+1})\frac{dF(\widetilde{c}_{t+1})}{F(\underline{c}_{t+1})} + (1-s_t(1-\rho_{t+1}))\mathcal{V}_{t+1}^U\right]. \tag{15}$$

A free entry condition applies for firms, which implies $\mathcal{V}_t^V = 0$ for all t. Thus, we obtain from (12) and (13) respectively that

$$E_t\Xi_{t,t+1}(1-\rho_{t+1})\int_0^{\underline{c}_t}\mathcal{V}_{t+1}^J(\widetilde{c}_{t+1})\frac{dF(\widetilde{c}_{t+1})}{F(\underline{c}_{t+1})} = \frac{\omega_t}{e_t}, \tag{16}$$

and

$$\mathcal{V}_t^J(\widetilde{c}_t) = p_t^m mpn_t - W_t(\widetilde{c}_t)h_t - A_{t-1}\kappa_c\widetilde{c}_t + \frac{\omega_t}{e_t}. \tag{17}$$

Firms and workers choose the real wage $W_t^n(\widetilde{c}_t)$ and hours h_t to maximize the Nash product:

$$\max_{W_t^n,h_t}(\mathcal{V}_t^E(\widetilde{c}_t) - \mathcal{V}_t^U)^{\varphi}(\mathcal{V}_t^J(\widetilde{c}_t))^{1-\varphi},$$

where the first term is the worker's surplus and the second is the firm's surplus, while $\varphi \in (0,1)$ is the worker's relative bargaining power. The first-order conditions for $W_t^n(\widetilde{c}_t)$ and h_t imply that

$$p_t^m\frac{\partial mpn_t}{\partial h_t} = \frac{g'(h_t)}{\Lambda_t}.$$

This equation implicitly defines the amount of hours per worker. It shows that in equilibrium the marginal productivity of an extra worker-hour is equal to the marginal rate of substitution between h_t and C_t. Now, the first-order condition for $W_t^n(\widetilde{c}_t)$ implies that

$$(1-\varphi)\left(\mathcal{V}_t^E(\widetilde{c}_t) - \mathcal{V}_t^U\right) = \varphi\mathcal{V}_t^J(\widetilde{c}_t). \tag{18}$$

Using (14)-(17) in (18), and using $s_t/e_t = v_t/u_t$ yields the wage equation of an individual worker:

$$W_t^n(\tilde{c}_t)h_t = \varphi\left[p_t^m mpn_t - A_{t-1}\kappa_c\tilde{c}_t + \omega_t\frac{v_t}{u_t}\right] + (1-\varphi)\left(b_t^u + \frac{g(h_t)}{\Lambda_t}\right). \tag{19}$$

It expresses the wage payment to the worker as a weighted average, according to the relative bargaining power of the worker and the firm, between the marginal product of the worker minus operating costs plus the cost of replacing the worker (weighted by the relative probability of finding a job and replacing the worker, i.e. labor market tightness), and the outside option of the worker.

The aggregate real Nash wage is the average of (19) over the distribution of idiosyncratic costs:

$$W_t^n h_t = \varphi\left[p_t^m mpn_t - H(\underline{c}_t) + \omega_t\frac{v_t}{u_t}\right] + (1-\varphi)\left(b_t^u + \frac{g(h_t)}{\Lambda_t}\right),$$

where $H(\underline{c}_t)$ is the average operating cost. In order to allow for some degree of nominal wage stickiness through indexation, following Hall (2005), we assume that the effective nominal wage paid to the worker is a weighted average of the inflation-indexed past nominal wage and the Nash wage, with weights $\varkappa_W \in [0,1)$ and $1-\varkappa_W$ respectively:[23]

$$P_tW_t = \varkappa_W\Gamma_{t-1}^W P_{t-1}W_{t-1} + (1-\varkappa_W)P_tW_t^n,$$

where Γ_t^W is a wage indexation variable that satisfies $\Gamma_t^W = (A_t/A_{t-1})^{\alpha_W}\pi_t^{\vartheta_W}\bar{\pi}^{1-\vartheta_W}$, where $\bar{\pi}$ is target inflation.[24] The critical threshold at which jobs are destroyed endogenously is implicitly defined by $\mathcal{V}_t^J(\underline{c}_t) = 0$.[25] Using this condition with (17) and (19), we obtain

$$\begin{aligned} A_{t-1}\kappa_c\underline{c}_t &= p_t^m mpn_t + \frac{1-(1-\varkappa_W)\varphi s_t}{1-(1-\varkappa_W)\varphi}\frac{\omega_t}{e_t} - \frac{\varkappa_W}{1-(1-\varkappa_W)\varphi}h_t\frac{\Gamma_{t-1}^W}{\pi_t}W_{t-1} \\ &\quad - \frac{(1-\varkappa_W)(1-\varphi)}{1-(1-\varkappa_W)\varphi}\left(b_t^u + \frac{g(h_t)}{\Lambda_t}\right). \end{aligned}$$

Note that a higher marginal product of the worker increases $\underline{c}_t$ (i.e. ρ_t^n decreases) while an increase in the worker's outside option decreases $\underline{c}_t$ (i.e. ρ_t^n increases).

[23] Hall (2005) considers real wage inertia while we consider nominal wage inertia with indexation to account for the importance of inflation indexation of nominal wages in many emerging market economies including Chile. In order to keep the model simple, we do not adopt a more sophisticated specification of wage stickiness as Gertler and Trigari (2009) and Gertler, Sala, and Trigari (2008) or Christiano, Eichenbaum, and Trabandt (2016).

[24] The parameter α_W controls whether wages are indexed to the stochastic trend ($\alpha_W = 1$), as is typically the case in models with Calvo wages, or not ($\alpha_W = 0$).

[25] The joint surplus of a match is given by $\mathcal{V}_t^S(\tilde{c}_t) = \mathcal{V}_t^J(\tilde{c}_t) + \mathcal{V}_t^E(\tilde{c}_t) - \mathcal{V}_t^U$. A match is endogenously separated whenever $\mathcal{V}_t^S(\tilde{c}_t) \leq 0$ which is equivalent to $\mathcal{V}_t^J(\tilde{c}_t) \leq 0$.

2.5 Fiscal and Monetary Policy

The government consumes an exogenous stream of final goods (G_t), pays unemployment benefits, levies lump-sum taxes, issues one-period bonds and receives a share of the income generated in the commodity sector. We assume for simplicity that the public asset position is completely denominated in domestic currency. Hence, the government satisfies the following period-by-period constraint

$$G_t + b_t^u u_t + r_t B_{t-1} = T_t + B_t + \chi p_t^{Co} Y_t^{Co}.$$

Monetary policy is carried out according to a Taylor rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\alpha_\pi} \left(\frac{Y_t/Y_{t-1}}{a_{t-1}}\right)^{\alpha_y}\right]^{1-\rho_R} \exp(\varepsilon_t^R),$$

where R is the monetary policy rate in the long-run, $\bar{\pi}$ is target inflation and ε_t^R is an n.i.d. shock that captures deviations from the rule.

2.6 Rest of the World

Foreign agents demand home composite goods and buy the domestic commodity production. There are no transaction costs or other barriers to trade. The structure of the foreign economy is identical to the domestic economy, but the domestic economy is assumed to be small relative to the foreign economy. The latter implies that the foreign producer price level P_t^{F*} is identical to the foreign consumption-based price index P_t^*. Further, let P_t^{H*} denote the price of home composite goods expressed in foreign currency. Given full tradability and competitive export pricing, the law of one price holds separately for home composite goods and the commodity good, i.e. $P_t^H = S_t P_t^{H*}$ and $P_t^{Co} = S_t P_t^{Co*}$. That is, domestic and foreign prices of both goods are identical when expressed in the same currency. Due to local currency pricing, a weak form of the law of one price holds for foreign composite goods, i.e. $P_t^F mc_t^F = S_t P_t^{F*}$ from (11). The real exchange rate rer_t therefore satisfies

$$rer_t = \frac{S_t P_t^*}{P_t} = \frac{S_t P_t^{F*}}{P_t} = \frac{P_t^F mc_t^F}{P_t} = p_t^F mc_t^F,$$

and the commodity price in terms of domestic consumption goods is given by

$$p_t^{Co} = \frac{P_t^{Co}}{P_t} = \frac{S_t P_t^{Co*}}{P_t} = \frac{S_t P_t^*}{P_t} p_t^{Co*} = p_t^F p_t^{Co*}.$$

We also have the relation $rer_t/rer_{t-1} = \pi_t^S \pi_t^* / \pi_t$,where $\pi_t^S = S_t/S_{t-1}$ is the gross rate of nominal exchange rate depreciation. Further, foreign demand for the home composite good X_t^{H*} is given by the schedule

$$X_t^{H*} = o^* \left(\frac{P_t^{H*}}{P_t^*} \right)^{-\eta^*} Y_t^*, \qquad o^* \in (0,1), \qquad \eta^* > 0,$$

where Y_t^* denotes foreign aggregate demand or GDP. Both Y_t^* and π_t^* evolve exogenously.

2.7 Aggregation and Market Clearing

Taking into account the market clearing conditions for the different markets, we can define the trade balance in units of final goods as

$$TB_t = p_t^H X_t^{H*} + rer_t p_t^{Co*} Y_t^{Co} - rer_t IMP_t.$$

Further, we define real GDP as follows:

$$Y_t \equiv C_t + I_t + G_t + X_t^{H*} + Y_t^{Co} - IMP_t.$$

Then, the GDP deflator (p_t^Y, expressed as a relative price in terms of the final consumption good) is implicitly defined as

$$p_t^Y Y_t = C_t + I_t + G_t + TB_t.$$

Finally, the net foreign asset position evolves according to

$$rer_t B_t^* = rer_t r_t^* B_{t-1}^* + TB_t - (1-\chi) rer_t p_t^{Co*} Y_t^{Co}.$$

2.8 Driving Forces

For each exogenous variable in the model, we assume a process of the form

$$\log(x_t/\bar{x}) = F_x \log(x_{t-1}/\bar{x}) + \varepsilon_t^x, \qquad F_x \in [0,1), \qquad \bar{x} > 0,$$

for $x = \{\varrho, \kappa, \rho^x, \varpi, z, a, \zeta^o, \zeta^u, R^*, \pi^*, p^{Co*}, y^{Co}, y^*, g\}$, where the ε_t^x are n.i.d. shocks. We further assume that the idiosyncratic shock $\widetilde{c}_t$ is log-normally distributed with mean 0 and standard deviation $\sigma_{\widetilde{c}}$.[26]

[26] Several alternative distributions of the idiosyncratic shock have been used in the literature. Mortensen and Pissarides (1994) use a uniform distribution on the interval $[-1, 1]$ for the idiosyncratic shock. Den Haan et al. (2000) and Walsh (2005) use a log-normal distribution with mean 0. Guerrieri (2008) considers shocks distributed according to uniform, Pareto and log-normal distributions and finds no significant difference. Similarly, Tortorice (2013) finds that there is very little difference when using the uniform distribution in comparison to the log-normal distribution. Hence, we simply follow most of the literature and use a log-normal distribution.

3 The Model with Calvo Wages

This section briefly describes the model with the standard labor market specification, in which employment varies only at the intensive margin (hours), the labor market always clears, and there is monopolistic wage setting à la Calvo, following Schmitt-Grohé and Uribe (2006, 2007). Most of the model is identical to the model with search frictions. The differences are discussed below.

3.1 Households

Expected discounted utility of a representative household is given by

$$E_t \sum_{s=0}^{\infty} \beta_{t+s}^{s} \varrho_{t+s} \left[\frac{1}{1-\sigma} \left(C_{t+s} - \varsigma C_{t+s-1} \right)^{1-\sigma} - \kappa A_{t+s-1}^{1-\sigma} \frac{h_{t+s}^{1+\phi}}{1+\phi} \right]. \tag{20}$$

The period-by-period budget constraint of the household is given by

$$C_t + I_t + B_t + rer_t B_t^* + T_t = W_t h_t + r_t B_{t-1} + rer_t r_t^* B_{t-1}^* + r_t^K K_{t-1} + \Sigma_t. \tag{21}$$

The household chooses C_t, I_t, K_t, B_t, and B_t^* to maximize (20) subject to (21) and the capital stock level, taking r_t, r_t^*, r_t^K, rer_t, T_t, Σ_t, B_{t-1}, B_{t-1}^* and K_{t-1} as given.

3.2 Labor Union

Following Schmitt-Grohé and Uribe (2006, 2007), labor decisions are made by a central authority, a union, which supplies labor monopolistically to a continuum of labor markets indexed by $i \in [0,1]$. Households are indifferent between working in any of these markets. In each market, the union faces a demand for labor given by $h_t(i) = [W_t^n(i)/W_t^n]^{-\epsilon_W}\, h_t^d$, where $W_t^n(i)$ denotes the nominal wage charged by the union in market i, W_t^n is an aggregate hourly wage index that satisfies $(W_t^n)^{1-\epsilon_W} = \int_0^1 W_t^n(i)^{1-\epsilon_W} di$, and h_t^d denotes aggregate labor demand by firms. The union takes W_t^n and h_t^d as given and, once wages are set, it satisfies all labor demand. In addition, the total number of hours allocated to the different labor markets must satisfy the resource constraint $h_t = \int_0^1 h_t(i) di$. Wage setting is subject to a Calvo-type problem, whereby each period the household (or union) can set its nominal wage optimally in a fraction $1-\theta_W$ of randomly chosen labor markets, and in the remaining markets, the past wage rate is indexed to a weighted product of past and steady state inflation with weights $\vartheta_W \in [0,1]$ and $1-\vartheta_W$.

3.3 Wholesale Goods of Variety j

Wholesale goods of variety j are produced according to the technology

$$Y_t^H(j) = z_t K_{t-1}(j)^\alpha [A_t h_t^d(j)]^{1-\alpha}, \qquad \alpha \in [0,1). \tag{22}$$

Firm j's profit is given by $\Pi_t^m(j) = p_t^m(j) Y_t^H(j) - r_t^K K_{t-1}(j) - W_t h_t^d(j)$. The firm chooses $K_{t-1}(j)$ and $h_t^d(j)$ to maximize $\Pi_t^m(j)$ subject to (22). From the labor market clearing conditions we then obtain that $h_t = h_t^d \Delta_t^W$ in equilibrium, where Δ_t^W is a wage dispersion term.

3.4 Driving Forces

The Calvo wages model features the same driving forces as the model with search frictions, with the exception of the exogenous separation shock ρ_t^x, which does not apply.

4 Parametrization Strategy

Our empirical strategy combines both calibrated and estimated parameters. The calibrated parameters and targeted steady state values are presented in Table 1. For most of the parameters not related with the search frictions we draw from related studies for Chile, as indicated in the table, while others are endogenously determined in steady state to target some first moments ($\bar{\pi}^*$, $\bar{\kappa}$, o^*, $\bar{g}$ and $\bar{y}^{Co}$). The parameters that deserve additional explanation are those related to the search frictions: ω (the vacancy posting cost), $\bar{b}$ (the unemployment benefit), $\mu_{\tilde{c}}$ and $\kappa_{\tilde{c}}$ (the parameters of the stochastic operating cost), ρ^x (the exogenous separation rate), and φ (the workers' bargaining weight). The values of those parameters are either chosen to match observed statistics and available evidence for Chile, or following related studies for other countries.

We derive the vacancy posting cost (ω) from the steady state calculations to match an average unemployment rate (u) of around 8 percent between 1987 and 2014.[27] The implied vacancy cost to GDP ratio is approximately 4 percent, which is close to the value in Trigari (2009). The unemployment benefit ($\bar{b}$) is set to 0 based on OECD data.[28] Following Cooley and Quadrini (1999), den Haan et al. (2000) and other related studies, we set the probability of filling a vacancy in steady state (e) to 0.7. We further fix the total separation rate in steady state (ρ) based on evidence reported by Jones and Naudon (2009), who calculate quarterly labor status transition probabilities from micro survey data for Chile and find a probability of changing status from employed to unemployed, $p^{E,U}$, of about 0.04 as well as a probability of

[27] The average unemployment rate over the sample period from 2001Q3 to 2015Q2 was also about 8 percent.
[28] See https://data.oecd.org/socialexp/public-unemployment-spending.htm.

Table 1: Calibrated Parameters and Targeted Steady State Values.

Parameter	Description	Value	Source
Search model			
u	Unemployment rate in st. st.	0.08	Average (1987-2014)
$\bar{b}$	Unemployment benefit	0	OECD data (% of GDP)
e	Firm matching rate	0.7	Den Haan et al. (2000)
ρ	Total separation rate	0.0755	Jones and Naudon (2009)
ρ^x	Exog. separation rate	$\frac{2}{3}\rho$	Den Haan et al. (2000)
$\mu_{\tilde{c}}$	Log-normal mean of $\tilde{c}$	0	Normalization
φ	Workers' bargaining weight	0.5	Related literature
Calvo wage model			
ϵ_W	E. o. s. wages	11	Medina and Soto (2007)
Common parameters			
σ	Inverse intertemporal e. o. s.	1	Medina and Soto (2007)
α	Capital share in production	1-0.66	Medina and Soto (2007)
δ	Capital depreciation	0.06/4	Medina and Soto (2007)
ϵ_H	E. o. s. domestic aggregate	11	Medina and Soto (2007)
ϵ_F	E. o. s. imported aggregate	11	Medina and Soto (2007)
α_W	Indexation parameter	1	Medina and Soto (2007)
o	Share of F in Y^C	0.32	Average (1987-2014)
χ	Gov. share in commodity sector	0.61	Average (1987-2014)
s^{tb}	Trade balance to GDP in st. st.	0.04	Average (1987-2014)
s^g	Gov. cons. to GDP in st. st.	0.11	Average (1987-2014)
s^{Co}	Commod. prod. to GDP in st. st.	0.10	Average (1987-2014)
$\bar{\pi}$	Inflation in st. st.	3% p.a.	Inflation target in Chile
p^H	Relative price of H in st. st.	1	Normalization
h	Hours per worker in st. st.	0.3	Normalization
$\bar{a}$	Long-run growth	2% p.a.	Albagli et al. (2015)
β	Subjective discount factor	0.9995	MPR approx. 5%
R^*	Foreign rate in st. st.	4.5% p.a.	Fuentes and Gredig (2008)
ξ	Country premium in st. st.	1.5% p.a.	Average (1987-2014)

Note: All rates are annualized figures.

changing status from unemployed to employed, $p^{U,E}$, of about 0.47. These probabilities imply a value for ρ of approximately 7.5 percent, which is at the lower end of the range of quarterly U.S. worker separation rates of 8 to 10 percent reported by Hall (1995) and the values typically used in the literature.[29] Following den Haan et al. (2000), the exogenous separation rate (ρ^x) is then set to two thirds times the total separation rate. We further normalize the log-normal mean of the firm's operating cost to 0 and derive the scaling parameter $\kappa_{\tilde{c}}$ from the steady state calculations in order to match the targeted value of ρ. The workers' bargaining weight (φ) is set to 0.5, following the related literature.

We also calibrate the parameters characterizing those exogenous processes for which we have a data counterpart. In particular, for g we use linearly detrended real government consumption, for y^{Co} we use linearly detrended real mining production in the copper sector, for R^* we use the 3-month U.S. dollar London Interbank Offered Rate, for y^* we use linearly detrended real GDP of commercial partners while for π^* we use CPI inflation (in dollars) of commercial partners (both trade-weighted), and for p^{Co*} we use the price of refined copper at the London Metals Exchange (in dollars) deflated by the same price index used to construct π^*.[30]

The other parameters of the model were estimated using Bayesian techniques, solving the model with a log-linear approximation around the non-stochastic steady state. The list of these parameters and the priors are described in columns one to five of Table 4.[31] For the models with search frictions and Calvo wages, the following observable variables were used (all for a sample from 2001Q3, when the inflation targeting regime is implemented in Chile, to 2016Q1): the growth rates of real GDP, private consumption and investment, the CPI inflation rate, the monetary policy rate, the multilateral real exchange rate, the growth rate of real wages, total hours worked (hours per worker times employment divided by the labor force) and the EMBI Chile (which we match by the endogenous component of the country premium ξ_t plus the observed shock to the country premium ζ_t^o). We also include as observables the variables used to estimate the exogenous processes previously described.[32] In addition, for the model including search frictions we also use as observable the unemployment rate.

Overall, we use 16 observed variables in the estimation. Our estimation strategy also includes i.i.d. measurement errors for all observables. The variance of the latter was set to 10% of the variance of the corresponding observables.

[29] The value of ρ is calculated from (14) which implies that $p^{E,U} = \rho(1 - p^{U,E})$ in steady state, such that $\rho = p^{E,U}/(1 - p^{U,E}) \approx 0.0755$.

[30] The data source is the Central Bank of Chile's statistical database; see `http://si3.bcentral.cl/Siete`.

[31] The prior means were set to represent (when available) the estimates of related papers for the Chilean economy (e.g. Medina and Soto, 2007).

[32] While the parameters of these exogenous processes were calibrated, including these variables in the data set is informative for the inference of the innovations associated with the other exogenous processes.

Table 2: Marginal Data Densities.

	Search		
	With ρ^x_t	Without ρ^x_t	Calvo W
$\log p(X^T\|\theta)$	-1344.26	-1464.07	-1368.52
$\log p(X^T \text{ without } u^T\|\theta)$	-1284.68	-1283.77	-1368.52

Note: X^T denotes the full data set, X^T without u^T the set excluding the unemployment rate. For the model with search frictions, we also compute the marginal likelihoods shutting down the exogenous separation shock (ρ^x_t). The marginal data densities are Laplace approximations at the mean of the posterior distribution.

5 Results

In this section, we first assess the goodness of fit of the model under the different labor market specifications, in order to understand if and how the presence of search and matching helps to improve the ability of the model to account for the dynamics observed in the data. We then discuss the differences in the inferred parameters and compare the variance decomposition to find out which shocks are the most important drivers of the dynamics under Calvo wages and under search frictions. In addition, we analyze the impulse responses to selected shocks to understand the propagation properties of the different labor market frictions. Finally, we compare the forecasting performance of the two model variants against each other and against reduced-form Bayesian vector autoregressive (BVAR) benchmark forecasting models.

5.1 Goodness of Fit

To have an overall measure of goodness of fit, we compute the marginal data density implied by the posterior distribution of the parameters for each model. But the marginal data densities for the estimation samples are not immediately comparable, since the model with search frictions is estimated with an additional observable variable—the unemployment rate. Moreover, the model with search frictions has an additional shock—that to the exogenous component of the separation rate, which may give this model additional degrees of freedom to match the data compared to the model with Calvo wages. We therefore compute, based on the posterior distribution for the model with search frictions, the marginal data density excluding the unemployment rate (u^T) from the full data set (x^T), and shutting down the shock to the exogenous component of the separation rate. [33]

The results in Table 2 show that the overall fit of the model with search frictions is signif-

[33] The marginal data densities were computed through the Laplace approximation at the mean of the posterior distribution of the parameters.

Table 3: Second Moments.

Variable	Description	s.d. (%)			AC order 1			Correl. with GDP		
		Data	Search	Calvo W	Data	Search	Calvo W	Data	Search	Calvo W
$\Delta \log Y$	GDP	1.00	1.40	1.22	0.24	0.15	0.51	1.00	1.00	1.00
$\Delta \log C$	Consumption	1.08	0.86	1.27	0.62	0.61	0.79	0.75	0.31	0.25
$\Delta \log I$	Investment	4.10	5.40	5.31	0.21	0.36	0.77	0.52	0.62	0.59
TB/GDP	Nom. trade balance	5.29	5.67	4.85	0.77	0.94	0.95	0.37	-0.16	-0.13
π	Inflation	0.69	0.65	1.35	0.60	0.44	0.89	-0.09	-0.16	0.31
R	MPR	0.42	0.49	1.46	0.88	0.92	0.98	-0.36	0.01	0.13
rer	Real exch. rate	5.17	7.17	15.43	0.75	0.81	0.95	-0.24	-0.10	-0.16
ξ	EMBIG Chile	0.15	0.27	0.26	0.83	0.95	0.95	-0.48	-0.04	-0.12
$\Delta \log W$	Real wage	0.58	0.60	1.04	0.36	0.32	0.71	0.05	0.44	0.51
$h \times n$	Total hours worked	1.87	3.47	7.84	0.73	0.94	0.90	0.08	0.31	0.08
u	Unemployment rate	1.43	2.82	–	0.96	0.93	–	0.09	-0.32	–

Note: The model moments are the theoretical moments at the posterior mean.

icantly better than the fit of the model with Calvo wages. This result holds for the data sets including the unemployment rate (see the first row of the table), as well as the data set without the unemployment rate (second row), and independently of whether the exogenous separation shock is active or not. The difference between the marginal data densities is largest, more than 80 log points, when the data set excludes the unemployment rate and the exogenous separation shock is shut down. According to the usual scales of interpretation, this constitutes strong to very strong evidence in favor of the model with search frictions.[34]

To obtain a more detailed view of which variables are better matched by each model, Table 3 reports the standard deviations, first-order autocorrelation coefficients, and correlations with GDP of selected variables implied by the posterior mean of the parameters, and compares these statistics with the corresponding empirical moments. In terms of the standard deviations shown in the third to fifth columns of the table, the model with search frictions matches most variables better (with exceptions including real GDP, private consumption, investment, and the EMBI). The model with Calvo wages grossly overstates the standard deviation of hours worked, real wages and the real exchange rate. Likely related to the latter, it also overstates the standard deviation of inflation and the monetary policy rate. The autocorrelation coefficients in the sixth to eighth columns of the table show that the model with search frictions matches the persistence of all variables better than the model with Calvo wages (the only exception is hours worked). Note that the model with Calvo wages overstates the persistence of all variables. Finally, regarding correlations with GDP (ninth to eleventh columns), both models seem to perform reasonably well.

Overall, this goodness-of-fit analysis yields as a main conclusion that the model with search

[34]See Jeffreys (1961) and Kass and Raftery (1995).

frictions performs significantly better than the model with Calvo wages in terms of fitting both labor market data and other macroeconomic data. We examine next which properties of the model with search frictions explain this difference.

5.2 Estimated Parameters and Dynamics

Columns six to nine in Table 4 display the posterior mean and the 90% highest posterior density intervals of the estimated parameters of the two model variants. We will comment on those parameters whose inference is different between the models to see how the different labor market specifications affect the results.

One important parameter whose estimated value is different between the two models is the inverse Frisch elasticity of hours worked (ϕ), whose posterior mean is almost 3.5 times higher in the model with search frictions. Hence, the intensive margin is less important in that model compared to the model with Calvo wages. This result is in line with the findings of Christiano, Trabandt, and Walentin (2011) and other calibrations of search and matching models with both margins of labor supply (e.g. Trigari, 2009).

In terms of the parameters related to the nominal rigidities, both models rely on relatively large degrees of wage stickiness, as reflected by a high Calvo parameter for wages (θ_W) or a high wage inertia parameter ($\varkappa_W$). However, the degree of indexation of nominal wages to past inflation (ϑ_W) is much larger in the model with Calvo wages. Also, that model has a significantly higher Calvo parameter for home prices (θ_H) as well as a relatively low estimated reaction of monetary policy to inflation (α_π). Taken together, those results imply that both wages and inflation tend to be highly persistent in that model (see Table 3).

Other parameters that differ significantly between the two models are related to real rigidities. In particular, the model with Calvo wages has a significantly higher degree of habit formation (ς) and a higher elasticity of investment adjustment costs (γ). This may explain the relatively large size (i.e. the estimated innovation standard deviation and autocorrelation coefficient) of the consumption preference shock (ϱ) and the investment-specific technology shock (ϖ) in that model. In addition, the model with Calvo wages seems to require a relatively large labor supply preference shock (κ).

It is also instructive to study how the different shocks explain aggregate fluctuations. To that end, Table 5 displays the unconditional variance decomposition obtained for each version of the model for selected variables, computed at the respective posterior mean. In the model with Calvo wages, investment-specific technology shocks are the dominant driving force for most variables, followed far behind by foreign shocks. On the other hand, in the model with search

Table 4: Estimated Parameters.

					Posterior			
			Prior		Search		Calvo W	
Param.	Description	Dist.	Mean	s.d.	Mean	90% HPDI	Mean	90% HPDI
υ	Wealth effect size	beta	0.5	0.25	0.123	[0.000, 0.249]	0.022	[0.000, 0.048]
ϕ	Inv. elast. of h	norm	2	2	6.292	[4.374, 8.269]	1.712	[0.432, 3.054]
ς	Habit formation	beta	0.7	0.1	0.755	[0.693, 0.819]	0.867	[0.807, 0.928]
ψ	Country prem. elast.	invg	0.01	Inf	0.005	[0.003, 0.007]	0.004	[0.002, 0.005]
η	E. o. s. H and F	invg	1.5	0.25	3.706	[2.978, 4.428]	1.630	[1.260, 2.001]
η^*	RER elast. of X^{H*}	invg	0.25	0.1	0.553	[0.358, 0.733]	0.188	[0.117, 0.256]
γ	Inv. adj. cost	norm	4	1.5	0.334	[0.106, 0.555]	5.017	[3.155, 6.895]
$\sigma_{\tilde{c}}$	s.d. of $\tilde{c}$	norm	0.1	0.05	0.191	[0.117, 0.260]	–	–
μ	Match elast.	beta	0.5	0.1	0.516	[0.375, 0.665]	–	–
$\varkappa_W$	Inertia of W	beta	0.5	0.15	0.965	[0.952, 0.979]	–	–
θ_W	Calvo prob. W	beta	0.75	0.1	–	–	0.777	[0.666, 0.881]
ϑ_W	Index. past infl. W	beta	0.5	0.15	0.219	[0.131, 0.309]	0.739	[0.589, 0.900]
θ_H	Calvo prob. H	beta	0.75	0.1	0.305	[0.207, 0.407]	0.802	[0.757, 0.845]
ϑ_H	Index. past infl. H	beta	0.5	0.15	0.550	[0.319, 0.796]	0.170	[0.067, 0.273]
θ_H	Calvo prob. F	beta	0.75	0.1	0.896	[0.867, 0.925]	0.692	[0.621, 0.763]
ϑ_F	Index. past infl. F	beta	0.5	0.15	0.463	[0.300, 0.619]	0.461	[0.227, 0.690]
ρ_R	MPR rule R_{t-1}	beta	0.75	0.1	0.798	[0.756, 0.842]	0.801	[0.757, 0.846]
α_π	MPR rule π_t	norm	1.5	0.1	1.605	[1.466, 1.743]	1.329	[1.209, 1.444]
α_y	MPR rule Δy_t	norm	0.125	0.05	0.128	[0.057, 0.199]	0.096	[0.016, 0.171]
ρ_ϱ	AC cons. pref. sh.	beta	0.75	0.1	0.728	[0.597, 0.865]	0.810	[0.696, 0.930]
ρ_κ	AC labor pref. sh.	beta	0.75	0.1	0.772	[0.642, 0.909]	0.673	[0.513, 0.837]
ρ_{ρ^x}	AC separation sh.	beta	0.75	0.1	0.749	[0.612, 0.888]	–	–
ρ_ϖ	AC inv. sh.	beta	0.75	0.1	0.752	[0.627, 0.872]	0.947	[0.918, 0.978]
ρ_z	AC temp. TFP sh.	beta	0.75	0.1	0.762	[0.635, 0.884]	0.578	[0.472, 0.685]
ρ_a	AC perm. TFP sh.	beta	0.375	0.1	0.497	[0.377, 0.617]	0.306	[0.170, 0.443]
ρ_{ζ^o}	AC obs. risk sh.	beta	0.75	0.1	0.877	[0.816, 0.937]	0.849	[0.769, 0.935]
ρ_{ζ^u}	AC unobs. risk sh.	beta	0.75	0.1	0.796	[0.689, 0.897]	0.796	[0.728, 0.865]
σ_ϱ	s.d. cons. pref. sh.	invg	0.01	Inf	0.023	[0.016, 0.030]	0.059	[0.034, 0.084]
σ_κ	s.d. labor pref. sh.	invg	0.01	Inf	0.032	[0.015, 0.048]	0.072	[0.012, 0.132]
σ_{ρ^x}	s.d. separation sh.	invg	0.01	Inf	0.158	[0.099, 0.230]	–	–
σ_ϖ	s.d. inv. shock	invg	0.01	Inf	0.013	[0.008, 0.017]	0.068	[0.048, 0.087]
σ_z	s.d. temp. TFP sh.	invg	0.01	Inf	0.005	[0.003, 0.007]	0.018	[0.014, 0.022]
σ_a	s.d. perm. TFP sh.	invg	0.01	Inf	0.011	[0.008, 0.013]	0.015	[0.012, 0.018]
σ_{ζ^o}	s.d. obs. risk sh.	invg	0.003	Inf	0.001	[0.001, 0.001]	0.001	[0.001, 0.001]
σ_{ζ^u}	s.d. unobs. risk sh.	invg	0.003	Inf	0.007	[0.003, 0.010]	0.007	[0.004, 0.009]
σ_R	s.d. MPR shock	invg	0.003	Inf	0.002	[0.002, 0.002]	0.002	[0.001, 0.002]

Note: The results are based on 500,000 draws from the posterior distribution using the Metropolis-Hastings (MH) algorithm, dropping the first 250,000 draws to achieve convergence. The average acceptation rate of the MH algorithm was approximately 25% for each model. HPDI are the highest posterior density intervals. The priors for the parameters ϕ and α_π were truncated at 0 and 1, respectively. The computations were conducted using Dynare 4.4.3.

Table 5: Variance Decomposition.

Variable	TFP trans. z	TFP perm. a	Inv. tech. ϖ	**Total tech. sh.**	Cons. pref. ϱ	Lab. sup. κ	**Total pref. sh.**	**MP rate** e^R	**Gov. cons.** g	Risk obs. ζ^o	Risk unobs. ζ^u	**Total risk sh.**	For. rate R^*	For. infl. π^*	Co. price p^{Co*}	For. dem. y^*	**Total for. sh.**	**Co. prod.** y^{Co}	**Exo. sep.** ρ^x
										A. Search									
y	7	43	7	57	1	3	4	1	0	0	8	8	17	4	6	0	27	1	2
c	0	46	1	47	4	0	4	0	0	0	7	7	7	9	27	0	42	0	0
i	3	10	15	27	1	1	2	3	0	1	21	22	30	6	8	0	44	0	1
TB/GDP	1	8	12	20	0	0	0	5	0	1	14	15	28	8	22	1	58	0	0
π	5	24	28	57	0	2	2	23	0	0	6	6	3	5	2	0	10	0	2
R	1	34	32	66	0	0	1	4	0	0	7	7	10	4	7	0	22	0	0
rer	0	5	3	8	0	0	0	2	0	2	48	50	19	17	4	0	40	0	0
ξ	0	7	4	11	0	0	0	2	0	38	2	40	25	12	9	0	47	0	0
w	1	60	1	61	0	0	1	0	0	0	7	7	13	6	12	0	31	0	0
$h \times n$	4	37	12	53	1	6	7	4	0	0	7	7	11	3	6	0	20	0	8
u	4	37	13	54	0	1	1	5	0	0	7	7	13	3	4	0	20	0	12
										B. Calvo W									
y	0	3	76	79	2	1	3	0	0	0	2	2	5	2	8	0	16	0	–
c	0	8	27	35	17	0	18	0	0	0	4	4	8	7	28	0	43	0	–
i	0	3	77	80	2	0	2	0	0	0	2	3	8	2	5	0	15	0	–
TB/GDP	0	2	24	26	3	0	3	0	0	0	6	7	16	7	40	0	63	1	–
π	2	2	71	75	2	0	3	2	0	0	10	10	6	3	2	0	10	0	–
R	0	2	80	82	2	0	3	1	0	0	5	5	5	1	2	0	9	0	–
rer	0	6	69	75	1	0	2	0	0	0	8	9	6	4	4	0	14	0	–
ξ	0	3	28	32	1	0	1	0	0	31	3	33	8	11	16	0	34	0	–
w	1	5	40	46	2	8	10	0	0	0	4	4	9	6	24	0	40	0	–
$h \times n$	13	3	50	65	4	3	7	1	0	0	3	3	8	4	12	0	24	0	–

Note: The table entries are the fraction of the unconditional theoretical variances at the posterior mean (in %) explained by the shocks.

frictions, permanent TFP shocks (i.e., shocks to trend growth) and foreign shocks are the most important driving forces for most variables.[35] The literature has found these to be the most important driving forces of aggregate fluctuations in small open emerging economies. Aguiar and Gopinath (2007) highlight the role of permanent TFP shocks, whereas numerous articles emphasize the role of foreign shocks.[36] Moreover, Justiniano and Preston (2010) show that it is difficult for DSGE models to replicate the importance of foreign shocks found, for example, in vector autoregressive analyses. Our results suggest that search frictions may contribute to mitigate this shortcoming of SOE DSGE models. Finally, note that the exogenous separation rate shock explains merely up to 12% of the variance of the unemployment rate.

To better understand how shocks are propagated, we examine next the estimated impulse responses to selected shocks. In particular, we analyze the responses to a foreign interest rate shock (a foreign shock), a permanent TFP shock (a supply shock), and a domestic monetary policy shock (a demand shock), which are shown in figures 1, 2 and 3, respectively. In each figure, we compare the estimated impulse responses from the model with search frictions (blue solid lines) to the responses if we shut down the endogenous separations, i.e. $\rho_t^n = 0$ for all t (red dashed lines), and the model with Calvo wages (green dash-dotted lines).[37]

Figure 1 shows the effects of a foreign interest rate shock. It generates a contraction in output, consumption and investment. In the model with search frictions and endogenous separations, the contraction in real GDP is driven by a decline in employment, and not in hours worked, as in the model with Calvo wages and the model with search frictions but no endogenous separations. That is, in the model with search frictions with endogenous separations, the extensive margin of labor supply is the relevant margin of adjustment: firms post less vacancies, dismiss workers, and the unemployment rate increases; the resulting slackness in the labor market leads to lower pressure on wage growth. In the model with Calvo wages, wages respond countercyclically, growing positively amidst the economic contraction. This may be due to the higher exchange rate pass-through into domestic prices and indexation in that model (the tighter international financial conditions lead to a depreciation of the exchange rate and, thus, to higher import prices).

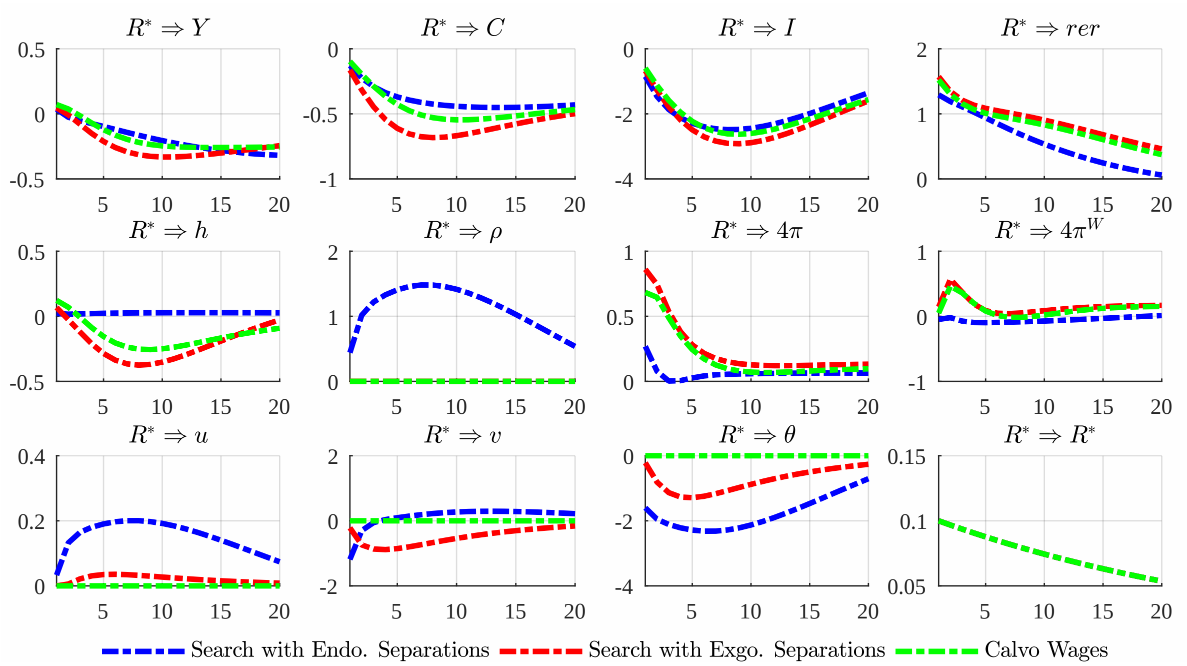
Figure 2 shows responses to a permanent TFP shock, i.e., a shock that induces a temporary increase in the *growth rate* of output and a permanent increase in its *level*. In the model with search frictions and endogenous separations, there is a substantial and long-lasting expansion

[35]Additionally, monetary policy shocks play an important role in inflation dynamics.

[36]See, for example, Fernández, Schmitt-Grohé, and Uribe (2017), Fernández, González, and Rodríguez (2017), Kose (2002), and Mendoza (1995).

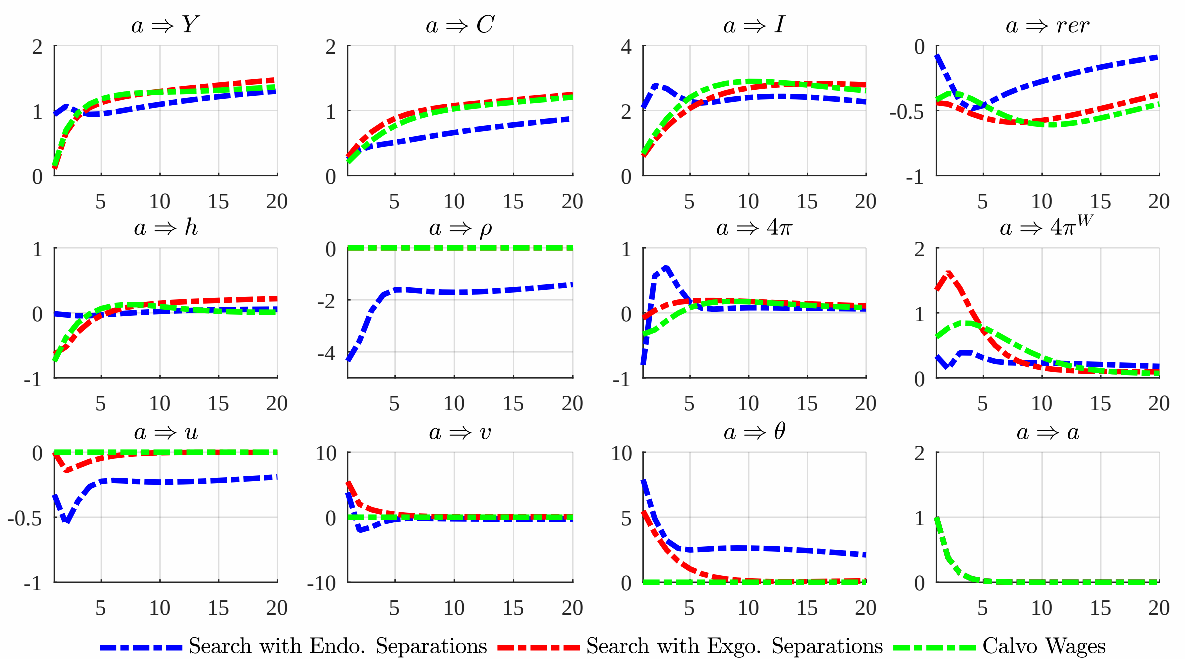
[37]To have the impulse responses comparable, the parameters associated to the shock process (persistence and volatility) are fixed at the prior mean, which is common across models.

Figure 1: Impulse Responses to a Foreign Interest Rate Shock (R^*).



Note: The blue solid lines correspond to the model with search frictions, the red dashed lines to the model without endogenous separations, and the green dash-dotted lines to the model with Calvo wages. In all cases the parameters associated to the shock process (persistence and volatility) are fixed at the prior mean, which is common across models. The variables are real GDP (Y), household consumption (C), investment (I), the real exchange rate (rer), hours per worker (h), the separation rate (ρ), annualized CPI inflation (4π), annualized nominal wage inflation ($4\pi^W$), the unemployment rate (u), vacancies (v), and labor market tightness (θ). All variables are expressed as percentage deviations from steady state.

Figure 2: Impulse Responses to a Permanent TFP Shock (a).



Note: See Figure 1.

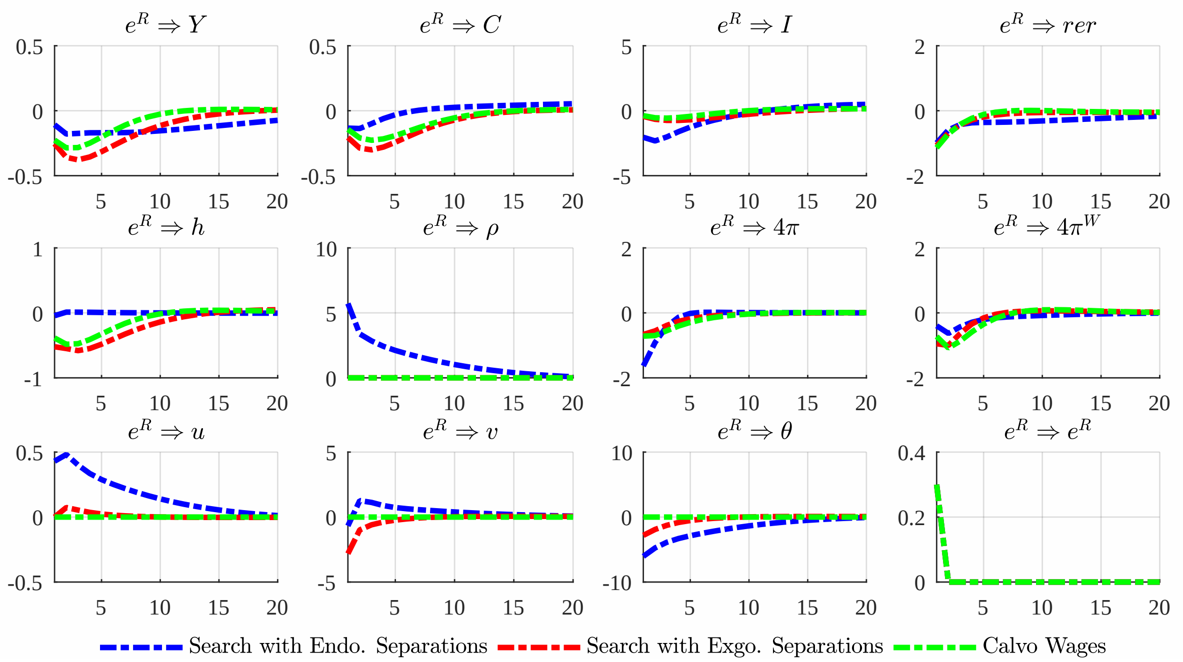
in employment (notice the decline in the unemployment rate); the labor market tightens, which puts upward pressure on wage growth. In the model with Calvo wages, hours worked decline. This countercyclical response in hours is due to a positive long-run wealth effect. Moreover, in the model with Calvo wages the shock generates a counterfactual negative relation between wages and hours.

Figure 3 shows responses to a domestic monetary policy shock. Again, the response of the extensive margin of labor supply is critical in the model with search frictions and endogenous separations: the unemployment rate increases in response to a contractionary shock, the labor market becomes more slack and wage growth declines. In the model with Calvo wages, the decline in output hinges on a decline in hours worked. This is also true for the model with search frictions but no endogenous separations.

Note that for all of the above shocks, the model with search and matching successfully replicates the so-called Beveridge curve, i.e., the empirically observed negative relation between vacancies and unemployment, at least on impact. (e.g. Krause and Lubik, 2007).

The above analysis leads us to two additional conclusions. First, search frictions and matching generate quantitatively relevant additional endogenous propagation properties of the model through variations of labor supply along the extensive margin, while the intensive margin be-

Figure 3: Impulse Responses to a Domestic Monetary Policy Shock (e^R).



Note: See Figure 1.

comes relatively less important. Second, the presence of endogenous separations is critical for the transmission of shocks by the labor market.[38]

5.3 Forecasting Performance

As a final step of the analysis, we conduct an out-of-sample forecasting experiment in order to judge how well the two models we analyze predict labor market data and other key variables such as output and inflation. For this experiment we estimated the model recursively and, for each estimation, forecasted the evolution of the observed variables several quarters ahead, starting in 2007Q1. Thus, the first estimation sample is 2001Q3-2006Q4 while the last sample is 2001Q3-2015Q1. The experiment is similar to that in Adolfson, Lindé, and Villani (2007), who evaluate the forecasting performance of a small open economy DSGE model for Swedish output, inflation and the monetary policy rate, and Christiano, Trabandt, and Walentin (2011), who extend that analysis to a model with search and matching and financial frictions. In addition to output, inflation and the monetary policy rate, we also analyze the forecasting performance of the two models for the real exchange rate, total hours worked and real wage growth.

Figure 4 shows the recursive forecasts for those variables from the model with search frictions

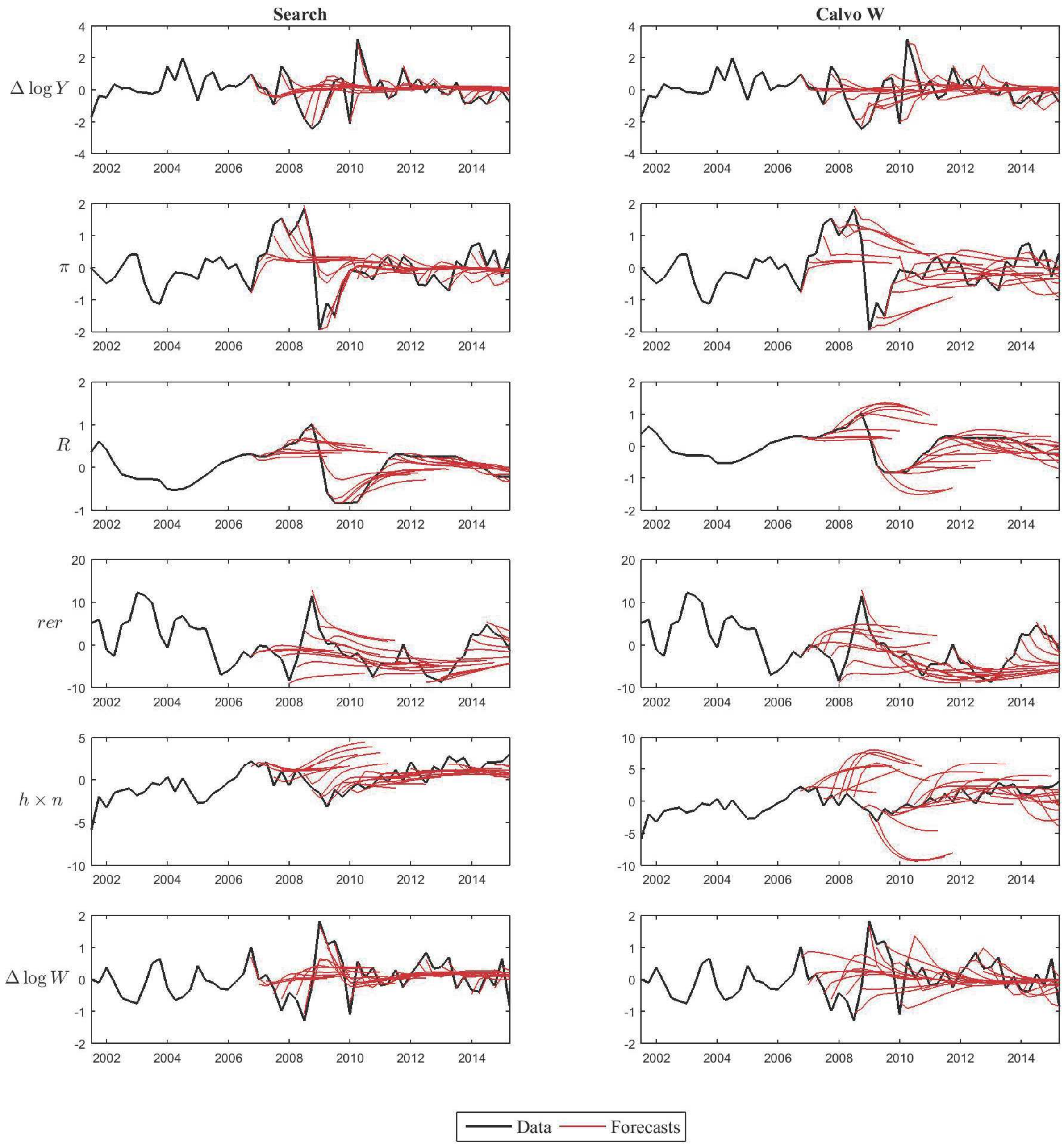
[38]These findings are well established in the literature (e.g. Den Haan et al., 2000; Trigari, 2009).

(left-hand side) and the model with Calvo wages (right-hand side). The results show that the model with search frictions does a better job than the model with Calvo wages in forecasting the evolution of all variables. In particular, real wages as well as total hours worked are predicted significantly better by the model with search frictions, but also inflation, the monetary policy rate and–with less but still noticeable differences–output and the real exchange rate. Note that the model with Calvo wages strongly overstates the persistence of inflation, in line with Section 5.1, which seems to be partly due to bad forecasts of real wage growth (given adequate predictions of the exchange rate).

To analyze the forecasting performance at different horizons, we also compute the root mean squared errors (RMSE) of the recursive forecasts at different horizons for the two models. As a benchmark, we compare the RMSE with those implied by three reduced-form BVARs that differ in the type of information that they incorporate. In particular, we estimate a basic model that includes real GDP growth, inflation, the monetary policy rate, the real exchange rate, total hours worked and real wage growth (BVAR1), as well as two bigger models that include all of the previous variables plus the growth rates of real private consumption and investment and real government consumption (BVAR2), or alternatively commercial partners' real GDP, the foreign interest rate, the copper price and commercial partners' inflation (BVAR3). All BVARs are estimated with a standard Minnesota-type prior following Doan, Litterman, and Sims (1983) and include four lags.

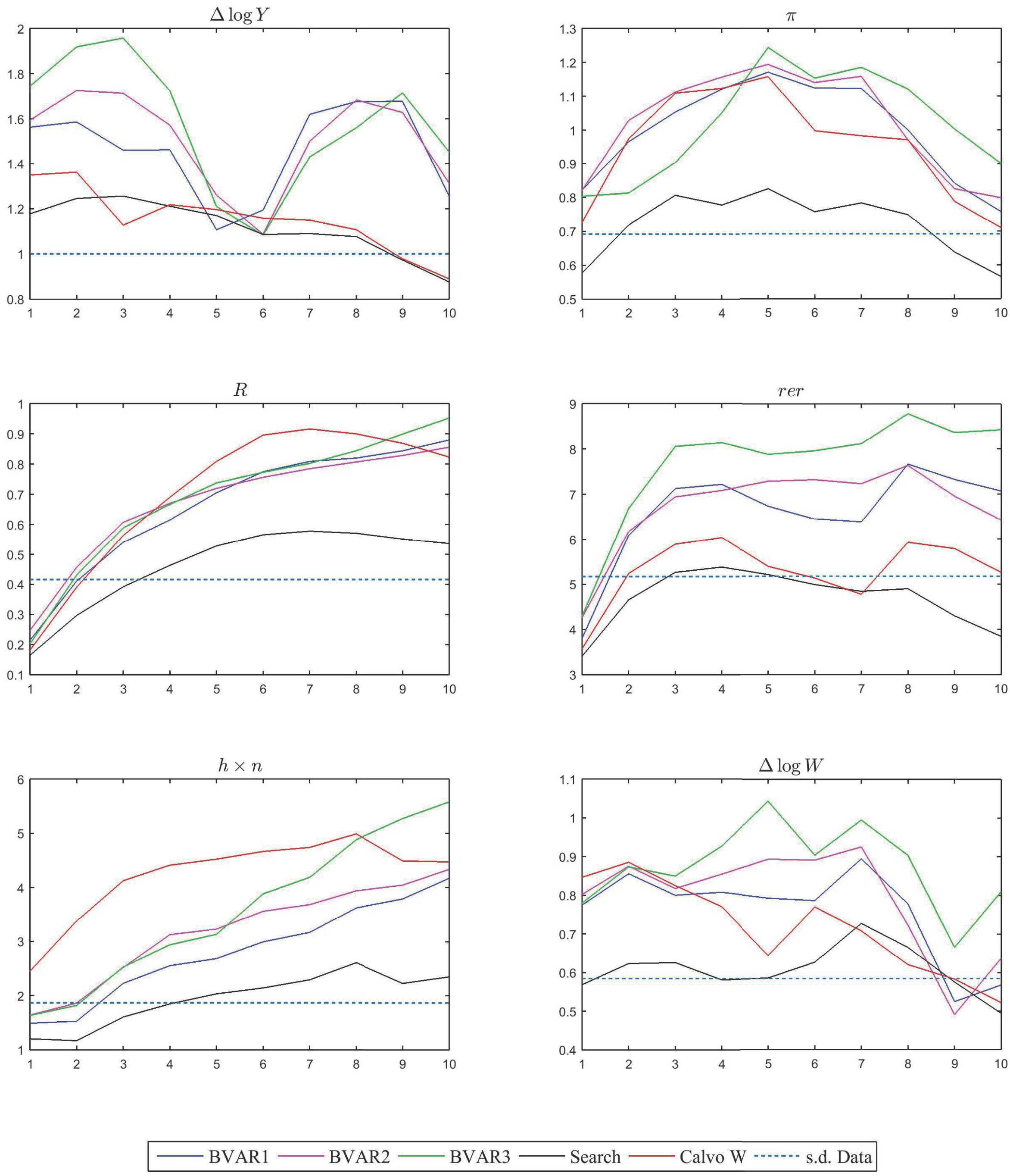
The RMSE for the different DSGE models and BVARs are shown in Figure 5. The results show that, while the DSGE model with Calvo wages predicts most variables roughly as well or better than the different BVARs, it is outperformed by the model with search frictions for almost all variables and horizons considered (1-10 quarters). Especially at short horizons, the RMSE for inflation, the monetary policy rate, the real exchange rate and hours worked from the model with search frictions are small (relative to the observed standard deviations). Hence, while the basic DSGE model with Calvo wages does perform relatively well compared to reduced-form empirical alternatives, which is a well-established finding in the literature (e.g. Smets and Wouters, 2003, 2007; Adolfson et al., 2007), the forecasting performance of the model is strongly improved by the inclusion of search frictions. The improved forecasting performance seems to be due to the fact that the model with search frictions can successfully explain the joint evolution of labor market data and other variables.

Figure 4: Recursive Out-of-Sample Forecasts.



Note: The black thick lines show the observed data while the red thin lines show the recursive forecasts. The forecasts are based on recursive estimations of the posterior mode of each model. The first estimation sample is 2001Q3-2006Q4 and the last estimation sample is 2001Q3-2015Q1.

Figure 5: Root Mean Squared Errors of Out-of-Sample Forecasts.



Note: See Figure 4. BVAR1 includes real GDP growth, inflation, the monetary policy rate, the real exchange rate, total hours worked and real wage growth. BVAR2 includes the variables from BVAR1 plus the growth rates of real private consumption and investment, and real government consumption. BVAR3 includes the variables from BVAR1 plus commercial partners' real GDP, the foreign interest rate, the copper price, and commercial partners' inflation. The variable transformations for the BVARs are the same as those adopted for the estimation of the DSGE models.

6 Conclusions

In this paper we have conducted a horse race of the standard labor market specification, which we call "Calvo wages," versus a search and matching specification with endogenous separations in an otherwise standard New Keynesian DSGE model for a small open economy. Our estimation results for Chilean data lead us to conclude that the search and matching specification "wins" by a wide margin as it significantly improves the model's ability to explain and predict both labor market data such as total hours worked and real wages and other macroeconomic variables such as output and inflation.

Our results thereby confirm several findings from previous studies and extend those findings to the context of an emerging market economy. In particular, similarly as Trigari (2009) we find that the model with search and matching explains variations in total hours mainly through the extensive margin of labor supply while the intensive margin is less important. Furthermore, similarly as Christiano, Trabandt, and Walentin (2011) and Krause, Lopez-Salido, and Lubik (2008), we find that labor supply shocks are relatively unimportant in the model with search and matching to explain the joint evolution of both labor market variables and other variables such as output and inflation. As in those studies, the presence of endogenous separations is key for the endogenous propagation of other structural shocks through the labor market.

However, unlike the main related study in the context of an NK-SOE model, i.e. Christiano, Trabandt, and Walentin (2011), we find that basic foreign shocks (in particular foreign interest rate shocks and shocks to commodity export prices) are a very important driving force in the model with search frictions. Compared to the model of Christiano, Trabandt, and Walentin (2011), we see the benefits of our approach mainly in its simplicity, being a relatively straightforward extension of an otherwise standard NK-SOE model to include search and matching with endogenous separations.

Overall, our results point to the usefulness of a search and matching specification with endogenous separations for medium-scale NK-SOE models. We expect they will be of special interest to economic modellers at central banks and other policy institutions who seek to improve the specification of DSGE models used for policy analysis and forecasting.

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A Equilibrium Conditions of the Search Model

The variables in uppercase that are not prices contain a unit root in equilibrium due to the presence of the non-stationary productivity shock A_t. We need to transform these variables to have a stationary version of the model. To do this, with the exceptions we enumerate below, lowercase variables denote the uppercase variable divided by A_{t-1} (e.g. $c_t \equiv \frac{C_t}{A_{t-1}}$). The only exception is the Lagrange multiplier Λ_t that is multiplied by A_{t-1}^{σ} (i.e. $\lambda_t \equiv \Lambda_t A_{t-1}^{\sigma}$), for it decreases along the balanced growth path.

The rational expectations equilibrium of the stationary version of the model model is the set of sequences

$$\{\lambda_t, c_t, h_t, \Theta_t, \widetilde{\chi}_t, i_t, k_t, r_t^K, q_t, y_t, y_t^C, y_t^F, y_t^H, x_t^F, x_t^H, x_t^{H*}, R_t, \xi_t, \pi_t, \pi_t^S, rer_t, p_t^H, \tilde{p}_t^H, p_t^F, \tilde{p}_t^F, p_t^Y,$$
$$mc_t^H, f_t^H, \Delta_t^H, mc_t^F, f_t^F, \Delta_t^F, b_t^*, imp_t, tb_t, n_t, u_t, v_t, s_t, e_t, p_t^m, w_t, w_t^n, \rho_t, \rho_t^n, \underline{c}_t, h_t^C, \gamma_t^W\}_{t=0}^{\infty},$$

(48 variables) such that for given initial values and exogenous sequences

$$\{\kappa_t, \varrho_t, \varpi_t, z_t, a_t, \rho_t^x, \zeta_t^o, \zeta_t^u, R_t^*, \pi_t^*, p_t^{Co*}, y_t^{Co}, y_t^*, g_t\}_{t=0}^{\infty},$$

and assuming

$$\widetilde{c}_t \sim \log N(0, \sigma_{\widetilde{c}}),$$

the following conditions are satisfied:

$$\lambda_t = \left(c_t - \varsigma \frac{c_{t-1}}{a_{t-1}}\right)^{-\sigma}, \tag{23}$$

$$p_t^m (1-\alpha)^2 z_t a_t^{1-\alpha} \left(\frac{k_{t-1}}{a_{t-1} n_t}\right)^{\alpha} = \Theta_t \kappa_t \frac{h_t^{\alpha+\phi}}{\lambda_t}, \tag{24}$$

$$\lambda_t = \frac{\beta}{a_t^{\sigma}} R_t E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\pi_{t+1}} \right\}, \tag{25}$$

$$\lambda_t = \frac{\beta}{a_t^{\sigma}} R_t^* \xi_t E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\pi_{t+1}^S \lambda_{t+1}}{\pi_{t+1}} \right\}, \tag{26}$$

$$y_t^C = \left[(1-o)^{\frac{1}{\eta}} \left(x_t^H\right)^{\frac{\eta-1}{\eta}} + o^{\frac{1}{\eta}} \left(x_t^F\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{27}$$

$$x_t^F = o \left(p_t^F\right)^{-\eta} y_t^C, \tag{28}$$

$$x_t^H = (1-o) \left(p_t^H\right)^{-\eta} y_t^C, \tag{29}$$

$$mc_t^H = \frac{p_t^m}{p_t^H}, \tag{30}$$

$$f_t^H = \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H y_t^H + \frac{\beta}{a_t^{\sigma-1}} \theta_H E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_t^{\vartheta_H} \pi^{1-\vartheta_H}}{\pi_{t+1}} \right)^{-\epsilon_H} \left(\frac{\tilde{p}_t^H}{\tilde{p}_{t+1}^H} \right)^{-\epsilon_H} \left(\frac{p_t^H}{p_{t+1}^H} \right)^{-1-\epsilon_H} f_{t+1}^H \right\}, \tag{31}$$

$$f_t^H = \left(\tilde{p}_t^H\right)^{1-\epsilon_H} y_t^H \left(\frac{\epsilon_H - 1}{\epsilon_H}\right) + \frac{\beta}{a_t^{\sigma-1}} \theta_H E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_t^{\vartheta_H} \pi^{1-\vartheta_H}}{\pi_{t+1}}\right)^{1-\epsilon_H} \left(\frac{\tilde{p}_t^H}{\tilde{p}_{t+1}^H}\right)^{1-\epsilon_H} \left(\frac{p_t^H}{p_{t+1}^H}\right)^{-\epsilon_H} f_{t+1}^H \right\}, \tag{32}$$

$$x_t^{H*} = o^* \left(\frac{p_t^H}{rer_t}\right)^{-\eta^*} y_t^*, \tag{33}$$

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\alpha_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{\alpha_y} \right]^{1-\rho_R} \exp(\varepsilon_t^R), \tag{34}$$

$$y_t^H \Delta_t^H = z_t \left(\frac{k_{t-1}}{a_{t-1}}\right)^{\alpha} (a_t h_t n_t)^{1-\alpha}, \tag{35}$$

$$1 = \theta_H \left(\frac{p_{t-1}^H}{p_t^H} \frac{\pi_{t-1}^{\vartheta_H} \pi^{1-\vartheta_H}}{\pi_t}\right)^{1-\epsilon_H} + (1-\theta_H) \left(\tilde{p}_t^H\right)^{1-\epsilon_H}, \tag{36}$$

$$y_t^H = x_t^H + x_t^{H*}, \tag{37}$$

$$y_t^C = c_t + i_t + g_t + n_t h_t^C + \omega v_t, \tag{38}$$

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^S \pi_t^*}{\pi_t}, \tag{39}$$

$$y_t = c_t + i_t + g_t + x_t^{H*} + y_t^{Co} - imp_t, \tag{40}$$

$$tb_t = p_t^H x_t^{H*} + rer_t p_t^{Co*} y_t^{Co} - rer_t imp_t, \tag{41}$$

$$rer_t b_t^* = rer_t \frac{b_{t-1}^*}{a_{t-1}\pi_t^*} R_{t-1}^* \xi_{t-1} + tb_t - (1-\chi) rer_t p_t^{Co*} y_t^{Co}, \tag{42}$$

$$\xi_t = \bar{\xi} \exp\left[-\psi \frac{rer_t b_t^* - rer \times b^*}{rer \times b^*} + \frac{\zeta_t^o - \zeta^o}{\zeta^o} + \frac{\zeta_t^u - \zeta^u}{\zeta^u} \right], \tag{43}$$

$$\Delta_t^H = (1-\theta_H) \left(\tilde{p}_t^H\right)^{-\epsilon_H} + \theta_H \left(\frac{p_{t-1}^H}{p_t^H} \frac{\pi_{t-1}^{\vartheta_H} \pi^{1-\vartheta_H}}{\pi_t}\right)^{-\epsilon_H} \Delta_{t-1}^H, \tag{44}$$

$$k_t = (1-\delta) \frac{k_{t-1}}{a_{t-1}} + \left[1 - \frac{\gamma}{2} \left(\frac{i_t}{i_{t-1}} a_{t-1} - \bar{a}\right)^2 \right] \varpi_t i_t, \tag{45}$$

$$q_t = \frac{\beta}{a_t^{\sigma}} E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\lambda_t} \left[r_{t+1}^K + q_{t+1}(1-\delta) \right] \right\}, \tag{46}$$

$$r_t^K = p_t^m \alpha \frac{y_t^H}{k_{t-1}} a_{t-1}, \tag{47}$$

$$\begin{aligned} \frac{1}{q_t} \quad = \quad & \left[1 - \frac{\gamma}{2} \left(\frac{i_t}{i_{t-1}} a_{t-1} - \bar{a}\right)^2 - \gamma \left(\frac{i_t}{i_{t-1}} a_{t-1} - \bar{a}\right) \frac{i_t}{i_{t-1}} a_{t-1} \right] u_t \\ & + \frac{\beta}{a_t^{\sigma}} \gamma E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\lambda_t} \frac{q_{t+1}}{q_t} \left(\frac{i_{t+1}}{i_t} a_t - \bar{a}\right) \left(\frac{i_{t+1}}{i_t} a_t\right)^2 u_{t+1} \right\}, \end{aligned} \tag{48}$$

$$p_t^Y y_t = c_t + i_t + g_t + tb_t, \tag{49}$$

$$1 = \theta_F \left(\frac{p^F_{t-1}}{p^F_t} \frac{\pi^{\vartheta_F}_{t-1} \pi^{1-\vartheta_F}}{\pi_t} \right)^{1-\epsilon_F} + (1-\theta_F) \left(\tilde{p}^F_t \right)^{1-\epsilon_F}, \tag{50}$$

$$y^F_t = x^F_t, \tag{51}$$

$$imp_t = y^F_t \Delta^F_t, \tag{52}$$

$$\Delta^F_t = (1-\theta_F) \left(\tilde{p}^F_t \right)^{-\epsilon_F} + \theta_F \left(\frac{p^F_{t-1}}{p^F_t} \frac{\pi^{\vartheta_F}_{t-1} \pi^{1-\vartheta_F}}{\pi_t} \right)^{-\epsilon_F} \Delta^F_{t-1}, \tag{53}$$

$$mc^F_t = rer_t / p^F_t, \tag{54}$$

$$f^F_t = \left(\tilde{p}^F_t \right)^{-\epsilon_F} y^F_t mc^F_t + \frac{\beta}{a^{\sigma-1}_t} \theta_F E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi^{\vartheta_F}_t \pi^{1-\vartheta_F}}{\pi_{t+1}} \right)^{-\epsilon_F} \left(\frac{\tilde{p}^F_t}{\tilde{p}^F_{t+1}} \right)^{-\epsilon_F} \left(\frac{p^F_t}{p^F_{t+1}} \right)^{-1-\epsilon_F} f^F_{t+1} \right\}, \tag{55}$$

$$f^F_t = \left(\tilde{p}^F_t \right)^{1-\epsilon_F} y^F_t \left(\frac{\epsilon_F - 1}{\epsilon_F} \right) + \frac{\beta}{a^{\sigma-1}_t} \theta_F E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi^{\vartheta_F}_t \pi^{1-\vartheta_F}}{\pi_{t+1}} \right)^{1-\epsilon_F} \left(\frac{\tilde{p}^F_t}{\tilde{p}^F_{t+1}} \right)^{1-\epsilon_F} \left(\frac{p^F_t}{p^F_{t+1}} \right)^{-\epsilon_F} f^F_{t+1} \right\}, \tag{56}$$

$$n_t = (1-\rho_t) \left(n_{t-1} + m \; v^{1-\mu}_{t-1} (u_{t-1})^{\mu} \right), \tag{57}$$

$$u_t = 1 - n_t, \tag{58}$$

$$s_t = m \; (v_t / u_t)^{1-\mu}, \tag{59}$$

$$e_t = m \; (v_t / u_t)^{-\mu}, \tag{60}$$

$$\frac{\omega}{e_t} = E_t \frac{\beta}{a^{\sigma-1}_t} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\lambda_t} (1-\rho_{t+1}) \left(p^m_{t+1} (1-\alpha) z_{t+1} a^{1-\alpha}_{t+1} \left(\frac{k_t}{a_t n_{t+1}} \right)^{\alpha} h^{1-\alpha}_{t+1} - h^C_{t+1} - w_{t+1} h_{t+1} + \frac{\omega}{e_{t+1}} \right), \tag{61}$$

$$\begin{aligned} w^n_t h_t = & \varphi \left[p^m_t (1-\alpha) z_t a^{1-\alpha}_t \left(\frac{k_{t-1}}{a_{t-1} n_t} \right)^{\alpha} h^{1-\alpha}_t - h^C_t + \omega \frac{v_t}{u_t} \right] \\ & + (1-\varphi) \left(\bar{b} + \frac{1}{\lambda_t} \left[\Theta_t \kappa_t \frac{h^{1+\phi}_t}{1+\phi} \right] \right). \end{aligned} \tag{62}$$

$$\pi_t w_t = \varkappa_W \gamma^W_{t-1} \frac{w_{t-1}}{a_{t-1}} + (1-\varkappa_W) \pi_t w^n_t, \tag{63}$$

$$\rho_t = \rho^x_t + (1-\rho^x_t) \rho^n_t. \tag{64}$$

Given the distribution assumption for $\widetilde{c}_t$, we have

$$\rho^n_t = 1 - F(\underline{c}_t) = 1 - \Phi \left(\frac{\ln \underline{c}_t}{\sigma_{\widetilde{c}}} \right), \tag{65}$$

where Φ is the standard normal c.d.f.

$$\kappa_c \underline{c}_t = p_t^m (1-\alpha) z_t a_t^{1-\alpha} \left(\frac{k_{t-1}}{a_{t-1} n_t}\right)^{\alpha} h_t^{1-\alpha} + \frac{1-(1-\varkappa_W)\varphi s_t}{1-(1-\varkappa_W)\varphi} \frac{\omega}{e_t}$$
$$- \frac{\varkappa_W}{1-(1-\varkappa_W)\varphi} h_t \frac{\gamma_{t-1}^W}{\pi_t} \frac{w_{t-1}}{a_{t-1}} - \frac{(1-\varkappa_W)(1-\varphi)}{1-(1-\varkappa_W)\varphi} \left(\bar{b} + \frac{1}{\lambda_t}\left[\Theta_t \kappa_t \frac{h_t^{1+\phi}}{1+\phi}\right]\right), \tag{66}$$

Given the distribution assumption for $\widetilde{c}_t$, we have

$$h_t^C = \kappa_{\widetilde{c}} \frac{\exp\left(\frac{\sigma_{\widetilde{c}}^2}{2}\right) \Phi\left(\frac{\ln \underline{c}_t - \sigma_{\widetilde{c}}^2}{\sigma_{\widetilde{c}}}\right)}{1-\rho_t^n}, \tag{67}$$

$$\gamma_t^W = a^{\alpha_W} \pi_t^{\vartheta_W} \bar{\pi}^{1-\vartheta_W}. \tag{68}$$

$$\Theta_t = \widetilde{\chi}_t (c_t - \varsigma \frac{c_{t-1}}{a_{t-1}})^{-\sigma}. \tag{69}$$

$$\widetilde{\chi}_t = \widetilde{\chi}_{t-1}^{1-\upsilon} \left(c_t - \varsigma \frac{c_{t-1}}{a_{t-1}}\right)^{\sigma\upsilon} \tag{70}$$

The exogenous processes are

$$\log(x_t/\bar{x}) = F_x \log(x_{t-1}/\bar{x}) + \varepsilon_t^x, \qquad F_x \in [0,1), \qquad \bar{x} > 0,$$

for $x = \{\varrho, \kappa, \rho^x, \varpi, z, a, \zeta^o, \zeta^u, R^*, \pi^*, p^{Co*}, y^{Co}, y^*, g\}$, where the ε_t^x are n.i.d. shocks.

B Steady State of the Search Model

We show how to compute the steady state for given values of h, u, $\rho = p^{E,U}/(1-p^{U,E})$, $s_{\rho_x} = \rho_x/\rho$, e, p^H, $s^{tb} = tb/\left(p^Y y\right)$, $s^g = g/\left(p^Y y\right)$ and $s^{Co} = rer \times p^{Co*} y^{Co}/\left(p^Y y\right)$. The parameters $\bar{\kappa}$, ω, $\kappa_{\widetilde{c}}$, ρ_x, $\bar{m}$, o^*, $\overline{p}^*$, $\bar{g}$ and $\bar{y}^{Co}$ are determined endogenously while the values of the remaining parameters are taken as given.

From the exogenous processes for ϱ_t, ϖ_t, z_t, a_t, y_t^{Co}, ζ_t^o, ζ_t^u, R_t^*, y_t^* and p_t^{Co*},

$$\varrho = \bar{\varrho},\ \varpi = \bar{\varpi},\ z = \bar{z},\ a = \bar{a},\ y^{Co} = \bar{y}^{Co},\ \zeta^o = \bar{\zeta}^o,\ \zeta^u = \bar{\zeta}^u,\ R^* = \bar{R}^*,\ y^* = \bar{y}^*,\ p^{Co*} = \bar{p}^{Co*},$$

From (43),

$$\xi = \bar{\xi}.$$

From (34),

$$\pi = \bar{\pi}.$$

From (25),

$$R = a^{\sigma}\pi/\beta.$$

From (48),

$$q = \varpi^{-1}.$$

From (46),

$$r^K = q\left(\frac{a^\sigma}{\beta} - 1 + \delta\right).$$

From (26),

$$\pi^S = a^\sigma \pi/(\beta R^* \xi).$$

From (39) and the exogenous process for π_t^*,

$$\pi^* = \bar{\pi}^* = \pi/\pi^S.$$

From (36), (50),

$$\tilde{p}^H = 1,\ \tilde{p}^F = 1.$$

From (44), (53),

$$\Delta^H = (\tilde{p}^H)^{-\epsilon_H},\ \Delta^F = (\tilde{p}^H)^{-\epsilon_F}.$$

From (31)-(32), (55)-(56),

$$mc^H = \frac{\epsilon_H - 1}{\epsilon_H}\tilde{p}^H,\ mc^F = \frac{\epsilon_F - 1}{\epsilon_F}\tilde{p}^F.$$

From (30),

$$p^m = p^H mc^H.$$

From (58),

$$n = 1 - u.$$

From $s_{\rho_x} = \rho_x/\rho$,

$$\rho_x = \rho s_{\rho_x}.$$

From (64),

$$\rho^n = \frac{\rho - \rho^x}{1 - \rho^x}.$$

From (65),

$$\underline{c} = \exp\left[\sigma_{\widetilde{c}}\Phi^{-1}\left(1 - \rho^n\right)\right].$$

From (57) and (60),

$$v = \frac{\rho n}{e\left(1 - \rho\right)}.$$

From (60) and the exogenous process for m_t,

$$m = \bar{m} = e(v/u)^\mu.$$

From (59),

$$s = m(v/u)^{1-\mu}.$$

From (35) and (47),

$$k = a^2 hn \left(\frac{\alpha p^m z}{\Delta^H r^K}\right)^{\frac{1}{1-\alpha}}.$$

From (35),

$$y^H = z\,(k/a)^{\alpha}\,(ahn)^{1-\alpha}/\Delta^H.$$

From (31),

$$f^H = mc^H\left(\tilde{p}^H\right)^{-\epsilon_H} y^H/(1-\beta a^{1-\sigma}\theta_H).$$

From (45),

$$i = k\left(\frac{1-(1-\delta)/a}{\varpi}\right).$$

From (27)-(29),

$$p^F = \left[\frac{1}{o} - \frac{1-o}{o}\left(p^H\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

From (54),

$$rer = mc^F p^F.$$

From (68),

$$\gamma^W = a^{\alpha_W}\pi.$$

From (69) and (70),

$$\Theta = 1.$$

From GDP equal to value added, equivalent to (49), (49) itself and (52),

$$\begin{aligned} p^Y y &= p^H y^H + p^Y y s^{Co} + p^F\left(1 - mc^F\Delta^F\right) o\left(p^F\right)^{-\eta} y^C - nh^C - \omega v \\ &= p^H y^H + p^Y y s^{Co} + p^F\left(1 - mc^F\Delta^F\right) o\left(p^F\right)^{-\eta}\left(c + i + g + nh^C + \omega v\right) - nh^C - \omega v \\ &= p^H y^H + p^Y y s^{Co} + p^F\left(1 - mc^F\Delta^F\right) o\left(p^F\right)^{-\eta}\left[p^Y y\left(1 - s^{tb}\right) + nh^C + \omega v\right] - nh^C - \omega v, \end{aligned}$$

or

$$p^Y y = \frac{p^H y^H + \left(nh^C + \omega v\right)\left[p^F\left(1 - mc^F\Delta^F\right) o\left(p^F\right)^{-\eta} - 1\right]}{1 - s^{Co} - p^F\left(1 - mc^F\Delta^F\right) o\left(p^F\right)^{-\eta}\left(1 - s^{tb}\right)}.$$

From $s^{tb} = tb/\left(p^Y y\right)$, $s^g = g/\left(p^Y y\right)$, $s^{Co} = rer \times p^{Co*} y^{Co}/\left(p^Y y\right)$ and the exogenous process for g_t,

$$tb = s^{tb} p^Y y,\; g = \bar{g} = s^g p^Y y,\; y^{Co} = \bar{y}^{Co} = s^{Co} p^Y y/\left(rer \times p^{Co*}\right).$$

From (29), (37), (38) and (49),

$$x^{H*} = y^H - (1-o)\left(p^H\right)^{-\eta}\left(p^Y y - tb + nh^C + \omega v\right).$$

From (41),

$$x^F = \left(p^H x^{H*} + rer \times p^{Co*} y^{Co} - tb\right)/rer.$$

From (28),

$$y^C = \left(x^F/o\right)\left(p^F\right)^{\eta}.$$

From (38),

$$c = y^C - g - i - nh^C - \omega v.$$

From (23),

$$\lambda = \left(c - \varsigma \frac{c}{a}\right)^{-\sigma}.$$

From (24) and the exogenous process for κ_t,

$$\kappa = \bar{\kappa} = \frac{p^m \lambda (1-\alpha)^2 z a^{1-2\alpha} (k/n)^{\alpha}}{\Theta h^{\alpha+\phi}}.$$

From (23), (66), (68),

$$\kappa_{\tilde{c}} = \frac{1}{\underline{c}} \left(\begin{array}{c} p^m (1-\alpha)\left(y^H \Delta^H/n\right) + \frac{1-(1-\varkappa_W)\varphi s}{1-(1-\varkappa_W)\varphi}\frac{\omega}{e} \\ -\frac{\varkappa_W}{1-(1-\varkappa_W)\varphi} a^{\alpha_W - 1} wh - \frac{(1-\varkappa_W)(1-\varphi)}{1-(1-\varkappa_W)\varphi}\left(\bar{b} + \frac{\Theta\kappa}{\lambda}\frac{h^{1+\phi}}{1+\phi}\right) \end{array} \right).$$

From (61),

$$\omega = e\left(\frac{a^{\sigma-1}}{\beta(1-\rho)} - 1\right)^{-1}\left(p^m (1-\alpha)\left(y^H \Delta^H/n\right) - h^C - wh\right).$$

From (23) and (62),

$$w = \frac{1}{h}\left(\frac{1-\varkappa_W}{1-\varkappa_W a^{\alpha_W-1}}\right)\left(\varphi\left[p^m(1-\alpha)\left(y^H\Delta^H/n\right) - h^C + \omega\frac{s}{e}\right] + (1-\varphi)\left(\bar{b} + \frac{\Theta\kappa}{\lambda}\frac{h^{1+\phi}}{1+\phi}\right)\right).$$

The last thirteen equations need to be solved numerically to obtain $p^Y y$, tb, g, y^{Co}, x^{H*}, x^F, y^C, c, λ, κ, $\kappa_{\tilde{c}}$, ω and w. From (70),

$$\tilde{\chi} = c^{\sigma}\left(1 - \frac{\varsigma}{a}\right)^{\sigma}.$$

From (63) and (68),

$$w^n = \left(\frac{1-\varkappa_W a^{\alpha_W-1}}{1-\varkappa_W}\right) w.$$

From (67),

$$h^C = \frac{\kappa_{\tilde{c}} \exp\left(\frac{\sigma_a^2}{2}\right)\Phi\left(\frac{\ln \underline{c} - \sigma_{\tilde{c}}^2}{\sigma_{\tilde{c}}}\right)}{1-\rho^n}.$$

From (33),

$$o^* = \left(x^{H*}/y^*\right)\left(p^H/rer\right)^{\eta^*}.$$

From (42),

$$b^* = \frac{tb - (1-\chi) rer \times p^{Co*} y^{Co}}{rer\left[1 - (R^* + \xi)/(\pi^* a)\right]}.$$

From (37),

$$x^H = y^H - x^{H*}.$$

From (51),

$$y^F = x^F.$$

From (55),

$$f^F = mc^F \left(\tilde{p}^F\right)^{-\epsilon_F} y^F / (1 - \beta a^{1-\sigma} \theta_F).$$

From (52),

$$imp = y^F \Delta^F.$$

From (40),

$$y = c + i + g + x^{H*} + y^{Co} - imp.$$

From (49),

$$p^Y = (c + i + g + tb) / y.$$

C Equilibrium Conditions of the Calvo Wages Model

The rational expectations equilibrium of the stationary version of the model is the set of sequences

$$\{\lambda_t, c_t, h_t, h_t^d, w_t, \tilde{w}_t, mc_t^W, f_t^W, \Delta_t^W, \gamma_t^W, i_t, k_t, r_t^K, q_t, y_t, y_t^C, y_t^F, y_t^H, x_t^F, x_t^H, x_t^{H*}, R_t,$$
$$\xi_t, \pi_t, \pi_t^S, rer_t, p_t^H, \tilde{p}_t^H, p_t^F, \tilde{p}_t^F, p_t^Y, p_t^m, mc_t^H, f_t^H, \Delta_t^H, mc_t^F, f_t^F, \Delta_t^F, b_t^*, imp_t, tb_t\}_{t=0}^{\infty},$$

(41 variables) such that for given initial values and exogenous sequences

$$\{\kappa_t, \varrho_t, \varpi_t, z_t, a_t, \zeta_t^o, \zeta_t^u, R_t^*, \pi_t^*, p_t^{Co*}, y_t^{Co}, y_t^*, g_t\}_{t=0}^{\infty},$$

conditions (23), (25)-(34), (36)-(37), (39)-(56), (68), and the following conditions are satisfied:

$$w_t mc_t^W = \kappa \frac{h_t^\phi}{\lambda_t}, \tag{71}$$

$$\begin{aligned} f_t^W &= mc_t^W \tilde{w}_t^{-\epsilon_W} h_t^d \\ &+ \frac{\beta}{a_t^{\sigma-1}} \theta_W E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\gamma_t^W}{a_t \pi_{t+1}} \right)^{-\epsilon_W} \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}} \right)^{-\epsilon_W} \left(\frac{w_t}{w_{t+1}} \right)^{-1-\epsilon_W} f_{t+1}^W \right\}, \end{aligned} \tag{72}$$

$$\begin{aligned} f_t^W &= \tilde{w}_t^{1-\epsilon_W} h_t^d \left(\frac{\epsilon_W - 1}{\epsilon_W} \right) \\ &+ \frac{\beta}{a_t^{\sigma-1}} \theta_W E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\gamma_t^W}{a_t \pi_{t+1}} \right)^{1-\epsilon_W} \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}} \right)^{1-\epsilon_W} \left(\frac{w_t}{w_{t+1}} \right)^{-\epsilon_W} f_{t+1}^W \right\}, \end{aligned} \tag{73}$$

$$1=(1-\theta_W)\tilde{w}_t^{1-\epsilon_W}+\theta_W\left(\frac{w_{t-1}}{w_t}\frac{\gamma_t^W}{a_{t-1}\pi_t}\right)^{1-\epsilon_W}, \tag{74}$$

$$\Delta_t^W=(1-\theta_W)\tilde{w}_t^{-\epsilon_W}+\theta_w\left(\frac{w_{t-1}}{w_t}\frac{\gamma_t^W}{a_{t-1}\pi_t}\right)^{-\epsilon_W}\Delta_{t-1}^W, \tag{75}$$

$$y_t^H\Delta_t^H=z_t\left(\frac{k_{t-1}}{a_{t-1}}\right)^{\alpha}(a_t h_t^d)^{1-\alpha}, \tag{76}$$

$$w_t=p_t^m\alpha\frac{y_t^H}{h_t^d}a_{t-1}, \tag{77}$$

$$h_t=h_t^d\Delta_t^W, \tag{78}$$

$$y_t^C=c_t+i_t+g_t. \tag{79}$$

D Steady State of the Calvo Wages Model

We solve for the steady state for given values of h, p^H, $s^{tb}=tb/\left(p^Y y\right)$, $s^g=g/\left(p^Y y\right)$ and $s^{Co}=rer\times p^{Co*}y^{Co}/\left(p^Y y\right)$. The parameters $\bar{\pi}^*$, κ, o^*, $\bar{g}$ and $\bar{y}^{Co}$ are determined endogenously while the values of the remaining parameters are taken as given. The following equations are added to the steady state of the model with search frictions.

From (74),

$$\tilde{w}=\left(\frac{1-\theta_W\left(\gamma^W/a\pi\right)^{1-\epsilon_W}}{1-\theta_W}\right)^{\frac{1}{1-\epsilon_W}}.$$

From (75),

$$\Delta^W=\frac{1-\theta_W}{1-\theta_w\left(\gamma^W/a\pi\right)^{-\epsilon_W}}\tilde{w}^{-\epsilon_W}.$$

From (72)-(73),

$$mc^W=\left(\frac{\epsilon_W-1}{\epsilon_W}\frac{1-\beta a^{1-\sigma}\left(\gamma^W/a\pi\right)^{-\epsilon_W}\theta_W}{1-\beta a^{1-\sigma}\left(\gamma^W/a\pi\right)^{1-\epsilon_W}\theta_W}\right)\tilde{w}.$$

From (78),

$$h^d=h/\Delta^W.$$

From (72),

$$f^W=\tilde{w}^{-\epsilon_W}h^d mc^W/\left(1-\beta a^{1-\sigma}\left(\gamma^W/a\pi\right)^{-\epsilon_W}\theta_W\right).$$

From (44), (48) and (77),

$$w=\left[\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}p^H mc^H z a^{1-\alpha}}{(r^K)^\alpha}\right]^{\frac{1}{1-\alpha}}.$$

From (44) and (77),

$$k=\frac{\alpha a w h^d}{(1-\alpha)\,r^K}.$$

From (76),

$$y^H=z\,(k/a)^\alpha\,(ah^d)^{1-\alpha}/\Delta^H.$$

From (79),

$$c = y^C - g - i.$$

From GDP equal to value added,

$$p^Y y = p^H y^H + p^Y y s^{Co} + p^F \left(1 - mc^F \Delta^F\right) o \left(p^F\right)^{-\eta} \left(1 - s^{tb}\right) p^Y y.$$

From (71),

$$\kappa = mc^W \lambda w / h^\phi.$$

The remaining equations are the same as in the model with search frictions, except for the equations corresponding to the labor market variables from the search model which are eliminated.